Y11 to Y12 Summer Independent Learning

June to August 2021

Part 1 is compulsory and part 2 is optional but highly recommended.
Please read the following instructions very carefully and ensure you label and collate all your work ready for checking in September.

For your first maths lesson please bring

- A large A4 folder with five subject dividers.
- These instructions with the tables filled in.
- Dated and titled work for each of the topics listed in Task 2.
- The two practice initial tests, fully marked and reviewed.
- A list of questions you need to ask prior to doing your initial test.

Part 1 Task 1 (compulsory) : Preparation Work

1. For each topic, work through video.
2. Complete worksheet **using the technique and layout** used in the video.
3. Make sure you title and date your work.
4. Mark and correct work.
5. Do improvement work as necessary.
6. Repeat for each topic.
7. Keep track by filling in the following table.
8. Collate your work for each topic together so it is easy to check in September. *(See point 3!)*

Topic	Video(s) *(Tick)*	Worksheet *(Tick)*	Details of Improvement Work Completed
B1 Indices			
B2 Surds			
B3 Quadratics			
B4 Simultaneous Equations			
B5 Inequalities			
E1 Triangle Geometry			

Part 1 Task 2 (compulsory)

1. Do Practice Initial Test 1 under exam conditions.
2. Mark and correct your test and identify any improvement work necessary.
3. Fill in the review sheet below.
4. Revisit relevant videos and worksheets.
5. Update review sheet with details of work completed.

Topic	Score	Improvement Work to Do	Tick
B1 Indices	11		
B2 Surds	10		
B3 Quadratics	49		
B4 Simultaneous Equations	11		
B5 Inequalities	11		
E1 Triangle Geometry	12		
Total	114		

6. Do Practice Initial Test 2 under exam conditions.
7. Mark and correct your test and identify any improvement work necessary.
8. Fill in the review sheet below.
9. Revisit relevant videos and worksheets.
10. Update review sheet with details of work completed.
11. Make a list of questions you need to ask prior to doing your initial test for real!

Topic	Score	Improvement Work to Do	Tick	Questions to ask…
B1 Indices	11			
B2 Surds	10			
B3 Quadratics	49			
B4 Simultaneous Equations	11			
B5 Inequalities	11			
E1 Triangle Geometry	12			
Total	114			

Part 2 (Optional but highly recommended optional task): GCSE Past Papers

1. Choose a paper and complete.
2. Use the mark scheme to mark and correct your work.
3. Add your score to the following table of results.
4. Repeat for all three papers!

Paper	Score	Areas for Improvement
AQA Paper 1 *(No Calculator)* Mark Scheme		
AQA Paper 2 Mark Scheme		
AQA Paper 3 Mark Scheme		

Note: You will just need to provide the above table in September.

Video hyperlinks

B1 Indices

https://youtu.be/1lThXgU08S0

https://youtu.be/v5bn4HZrmQs

https://youtu.be/W0h4rHj88ys

B2 Surds

https://youtu.be/jHelde32YtI

B3 Quadratics

https://youtu.be/Pziws8ojnlk

https://youtu.be/sn_joGVj15w

https://youtu.be/kk7p6hjn7hQ

https://youtu.be/toIqbX_NXHo

B4 Simultaneous Equations

https://youtu.be/4SRtwS5unwE

B5 Inequalities

https://youtu.be/wDut-In_7Wg

E1 Triangle Geometry

https://youtu.be/uVl6TAb0vBg

Topic: B1 Indices **Basic Skills videos:**

https://youtu.be/1lThXgU08S0

https://youtu.be/v5bn4HZrmQs

https://youtu.be/W0h4rHj88ys

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1. Evaluate

a 3^{-2} **b** $(\frac{2}{5})^{0}$ **c** $(-2)^{-6}$ **d** $(\frac{1}{6})^{-2}$ **e** $(1\frac{1}{2})^{-3}$ **f** $9^{\frac{1}{2}}$

2. Work out

a $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$ **b** $16^{\frac{1}{4}} + 25^{\frac{1}{2}}$ **c** $8^{-\frac{1}{3}} \div 36^{\frac{1}{2}}$ **d** $(-64)^{\frac{1}{3}} \times 9^{\frac{3}{2}}$

e $(\frac{1}{3})^{-2} - (-8)^{\frac{1}{3}}$ **f** $(\frac{1}{25})^{\frac{1}{2}} \times (\frac{1}{4})^{-2}$ **g** $81^{\frac{3}{4}} - (\frac{1}{49})^{-\frac{1}{2}}$ **h** $(\frac{1}{27})^{-\frac{1}{3}} \times (\frac{4}{9})^{-\frac{3}{2}}$

3. Solve each equation.

a $x^{\frac{1}{2}} = 6$ **b** $x^{\frac{1}{3}} = 5$ **c** $x^{-\frac{1}{2}} = 2$ **d** $x^{-\frac{1}{4}} = \frac{1}{3}$

4. Express in the form x^k

a $\sqrt{x}$ **b** $\frac{1}{\sqrt[3]{x}}$ **c** $x^2 \times \sqrt{x}$ **d** $\frac{\sqrt[4]{x}}{x}$

5. Express each of the following in the form ax^b, where a and b are rational constants.

a $\frac{4}{\sqrt{x}}$ **b** $\frac{1}{2x}$ **c** $\frac{3}{4x^3}$ **d** $\frac{1}{(3x)^2}$ **e** $\frac{2}{5\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt{9x^3}}$

6. Express in the form 2^k

a 8^2 **b** $(\frac{1}{4})^{-2}$ **c** $(\frac{1}{2})^{\frac{1}{3}}$ **d** $16^{-\frac{1}{6}}$ **e** $8^{\frac{2}{3}}$ **f** $(\frac{1}{32})^{-3}$

Advanced Skills

1. Express each of the following in the form 3^y, where y is a function of x.

a 9^x **b** 81^{x+1} **c** $27^{\frac{x}{4}}$ **d** $(\frac{1}{3})^x$ **e** 9^{2x-1} **f** $(\frac{1}{27})^{x+2}$

2. Given that $y = 2^x$, express each of the following in terms of y.

a 2^{x+1} **b** 2^{x-2} **c** 2^{2x} **d** 8^x **e** 2^{4x+3} **f** $(\frac{1}{2})^{x-3}$

3. Find the value of x such that

a $2^x = 64$ **b** $5^{x-1} = 125$ **c** $3^{x+4} - 27 = 0$ **d** $8^x - 2 = 0$

e $3^{2x-1} = 9$ **f** $16 - 4^{3x-2} = 0$ **g** $9^{x-2} = 27$ **h** $8^{2x+1} = 16$

4. Solve each equation.

a $2^{x+3} = 4^x$ **b** $5^{3x} = 25^{x+1}$ **c** $9^{2x} = 3^{x-3}$ **d** $16^x = 4^{1-x}$

e $4^{x+2} = 8^x$ **f** $27^{2x} = 9^{3-x}$ **g** $6^{3x-1} = 36^{x+2}$ **h** $8^x = 16^{2x-1}$

5. Solve the equation

$$25^x = 5^{4x+1}.$$

6. Given that $x = 2^{t-1}$ and $y = 2^{3t}$,

a find expressions in terms of t for

i xy **ii** $2y^2$

b Hence, or otherwise, find the value of t for which

$$2y^2 - xy = 0.$$

Exam Questions *(OCR/MEI C1 Questions)*

1. **Jan 05 Q5**

Find the value of the following.

(i) $\left(\frac{1}{3}\right)^{-2}$ [2]

(ii) $16^{\frac{3}{4}}$ [2]

2. **June 05 Q6**

Simplify the following.

(i) a^0 [1]

(ii) $a^6 \div a^{-2}$ [1]

(iii) $(9a^6b^2)^{-\frac{1}{2}}$ [3]

3. **June 06 Q9**

Simplify the following.

(i) $\dfrac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$ [2]

(ii) $\dfrac{12(a^3b^2c)^4}{4a^2c^6}$ [3]

4. **Jan 07 Q6**

Find the value of each of the following, giving each answer as an integer or fraction as appropriate.

(i) $25^{\frac{3}{2}}$ [2]

(ii) $\left(\dfrac{7}{3}\right)^{-2}$ [2]

5. **June 07 Q5**

(i) Find a, given that $a^3 = 64x^{12}y^3$. [2]

(ii) Find the value of $\left(\dfrac{1}{2}\right)^{-5}$. [2]

Answers – Basic Skills

1.

a $= \frac{1}{3^2} = \frac{1}{9}$ **b** $= 1$ **c** $= \frac{1}{(-2)^6} = \frac{1}{64}$

d $= 6^2 = 36$ **e** $= (\frac{3}{2})^{-3} = (\frac{2}{3})^3 = \frac{8}{27}$ **f** $= \sqrt{9} = 3$

2.

a $= \sqrt{4} \times \sqrt[3]{27}$
$= 2 \times 3 = 6$

b $= \sqrt[4]{16} + \sqrt{25}$
$= 2 + 5 = 7$

c $= \frac{1}{\sqrt[3]{8}} \div \sqrt{36}$
$= \frac{1}{2} \div 6 = \frac{1}{12}$

d $= \sqrt[3]{-64} \times (\sqrt{9})^3$
$= -4 \times 27 = -108$

e $= 3^2 - \sqrt[3]{-8}$
$= 9 - (-2) = 11$

f $= \sqrt{\frac{1}{25}} \times 4^2$
$= \frac{1}{5} \times 16 = \frac{16}{5}$ or $3\frac{1}{5}$

g $= (\sqrt[4]{81})^3 - \sqrt{49}$
$= 27 - 7 = 20$

h $= \sqrt[3]{27} \times (\sqrt{\frac{9}{4}})^3$
$= 3 \times \frac{27}{8} = \frac{81}{8}$ or $10\frac{1}{8}$

3.

a $x = 6^2 = 36$

b $x = 5^3 = 125$

c $x^{\frac{1}{2}} = \frac{1}{2}$
$x = (\frac{1}{2})^2 = \frac{1}{4}$

d $x^{\frac{1}{4}} = 3$
$x = 3^4 = 81$

4.

a $= x^{\frac{1}{2}}$ **b** $= x^{-\frac{1}{3}}$ **c** $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$ **d** $= \frac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$

e $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$ **f** $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$ **g** $= (x^{\frac{1}{2}})^5 = x^{\frac{5}{2}}$ **h** $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$

5.

a $4x^{-\frac{1}{2}}$ **b** $\frac{1}{2}x^{-1}$ **c** $\frac{3}{4}x^{-3}$ **d** $\frac{1}{9}x^{-2}$ **e** $\frac{2}{5}x^{-\frac{1}{3}}$ **f** $\frac{1}{3}x^{-\frac{3}{2}}$

6.

a $= (2^3)^2 = 2^6$ **b** $= (2^{-2})^{-2} = 2^4$ **c** $= (2^{-1})^{\frac{1}{3}} = 2^{-\frac{1}{3}}$

d $= (2^4)^{-\frac{1}{6}} = 2^{-\frac{2}{3}}$ **e** $= (2^3)^{\frac{2}{5}} = 2^{\frac{6}{5}}$ **f** $= (2^{-5})^{-3} = 2^{15}$

Answers – Advanced Skills

1.

a $= (3^2)^x = 3^{2x}$ **b** $= (3^4)^{x+1} = 3^{4x+4}$ **c** $= (3^3)^{\frac{4}{4}} = 3^{\frac{3}{4}x}$

d $= (3^{-1})^x = 3^{-x}$ **e** $= (3^2)^{2x-1} = 3^{4x-2}$ **f** $= (3^{-3})^{x+2} = 3^{-3x-6}$

2.

a $= 2 \times 2^x = 2y$ **b** $= 2^{-2} \times 2^x = \frac{1}{4}y$ **c** $= (2^x)^2 = y^2$

d $= (2^3)^x = 2^{3x} = (2^x)^3 = y^3$ **e** $= 2^3 \times 2^{4x} = 8y^4$ **f** $= (2^{-1})^{x-3} = 2^3 \times 2^{-x} = \frac{8}{y}$

3.

a $2^x = 2^6$
$x = 6$

b $5^{x-1} = 5^3$
$x - 1 = 3$
$x = 4$

c $3^{x+4} = 27 = 3^3$
$x + 4 = 3$
$x = -1$

d $(2^3)^x = 2^{3x} = 2$
$3x = 1$
$x = \frac{1}{3}$

e $3^{2x-1} = 3^2$
$2x - 1 = 2$
$x = \frac{3}{2}$

f $16 = 4^2 = 4^{3x-2}$
$2 = 3x - 2$
$x = \frac{4}{3}$

g $(3^2)^{x-2} = 3^{2x-4} = 3^3$
$2x - 4 = 3$
$x = \frac{7}{2}$

h $(2^3)^{2x+1} = 2^{6x+3} = 2^4$
$6x + 3 = 4$
$x = \frac{1}{6}$

4.

a $2^{x+3} = (2^2)^x = 2^{2x}$
$x + 3 = 2x$
$x = 3$

b $5^{3x} = (5^2)^{x+1} = 5^{2x+2}$
$3x = 2x + 2$
$x = 2$

c $(3^2)^{2x} = 3^{4x} = 3^{x-3}$
$4x = x - 3$
$x = -1$

d $(4^2)^x = 4^{2x} = 4^{1-x}$
$2x = 1 - x$
$x = \frac{1}{3}$

e $(2^2)^{x+2} = (2^3)^x$
$2^{2x+4} = 2^{3x}$
$2x + 4 = 3x$
$x = 4$

f $(3^3)^{2x} = (3^2)^{3-x}$
$3^{6x} = 3^{6-2x}$
$6x = 6 - 2x$
$x = \frac{3}{4}$

g $6^{3x-1} = (6^2)^{x+2}$
$6^{3x-1} = 6^{2x+4}$
$3x - 1 = 2x + 4$
$x = 5$

h $(2^3)^x = (2^4)^{2x-1}$
$2^{3x} = 2^{8x-4}$
$3x = 8x - 4$
$x = \frac{4}{5}$

5.

$25^x = (5^2)^x = 5^{4x+1}$
$5^{2x} = 5^{4x+1}$
$2x = 4x + 1$
$x = -\frac{1}{2}$

6.

a i $xy = 2^{t-1} \times 2^{3t} = 2^{4t-1}$

ii $2y^2 = 2 \times (2^{3t})^2 = 2 \times 2^{6t} = 2^{6t+1}$

b $2^{6t+1} - 2^{4t-1} = 0$

$2^{6t+1} = 2^{4t-1}$

$6t + 1 = 4t - 1$

$t = -1$

Indices Exam Questions Solutions

1. Jan 05 Q5

(i) $\left(\frac{1}{3}\right)^{-2}$

$= \left(\frac{3}{1}\right)^{2}$

$= 9$

(ii) $16^{3/4}$

$= (16^{1/4})^{3}$

$= 2^{3}$

$= 8$

2. Jan 05 Q6

(i) $a^{0} = 1$ (ii) $a^{6} \div a^{-2} = a^{8}$

(iii) $(9a^{6}b^{2})^{-1/2} = \frac{1}{3}a^{-3}b^{-1}$

or $\frac{1}{3a^{3}b}$

3. June 06 Q9

(i) $\frac{16^{1/2}}{81^{3/4}}$

$= \frac{4}{(81^{1/4})^{3}}$

$= \frac{4}{3^{3}}$

$= \frac{4}{27}$

(ii) $\frac{12(a^{3}b^{2}c)^{4}}{4a^{2}c^{6}}$

$= \frac{12a^{12}b^{8}c^{4}}{4a^{2}c^{6}}$

$= 3a^{10}b^{8}c^{-2}$

or $\frac{3a^{10}b^{8}}{c^{2}}$

4. Jan 07 Q6

(i) $25^{3/2} = (25^{1/2})^{3}$

$= 5^{3}$

$= 125$

(ii) $\left(\frac{7}{3}\right)^{-2} = \left(\frac{3}{7}\right)^{2}$

$= \frac{9}{49}$

5. June 07 Q5

(i) $a^{3} = 64x^{12}y^{3}$

$a = (64x^{12}y^{3})^{1/3}$

$a = 4x^{4}y$

(ii) $\left(\frac{1}{2}\right)^{-5}$

$= \left(\frac{2}{1}\right)^{5}$

$= 32$

Topic: B2 Surds **Basic Skills**

https://youtu.be/jHelde32YtI

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1. Evaluate

a $\sqrt{49}$ **b** $\sqrt{121}$ **c** $\sqrt{\frac{1}{9}}$ **d** $\sqrt{\frac{4}{25}}$ **e** $\sqrt{0.01}$ **f** $\sqrt{0.09}$

2. Simplify

a $\sqrt{7} \times \sqrt{7}$ **b** $4\sqrt{5} \times \sqrt{5}$ **c** $(3\sqrt{3})^2$ **d** $(\sqrt{6})^4$

3. Simplify

a $\sqrt{12}$ **b** $\sqrt{28}$ **c** $\sqrt{80}$ **d** $\sqrt{27}$ **e** $\sqrt{24}$ **f** $\sqrt{128}$

4. Simplify

a $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$

5. Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{2}{\sqrt{3}}$ **c** $\frac{1}{\sqrt{8}}$ **d** $\frac{14}{\sqrt{7}}$ **e** $\frac{3\sqrt{2}}{\sqrt{3}}$ **f** $\frac{\sqrt{5}}{\sqrt{15}}$

6. Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2}+1}$ **b** $\frac{4}{\sqrt{3}-1}$ **c** $\frac{1}{\sqrt{6}-2}$ **d** $\frac{3}{2+\sqrt{3}}$

Advanced Skills

1. Simplify

a $(\sqrt{5}+1)(2\sqrt{5}+3)$ **b** $(1-\sqrt{2})(4\sqrt{2}-3)$ **c** $(2\sqrt{7}+3)^2$

2. Simplify

a $\sqrt{8} + \frac{6}{\sqrt{2}}$ **b** $\sqrt{48} - \frac{10}{\sqrt{3}}$ **c** $\frac{6-\sqrt{8}}{\sqrt{2}}$

d $\frac{\sqrt{45}-5}{\sqrt{20}}$ **e** $\frac{1}{\sqrt{18}} + \frac{1}{\sqrt{32}}$ **f** $\frac{2}{\sqrt{3}} - \frac{\sqrt{6}}{\sqrt{72}}$

3. Solve the equation

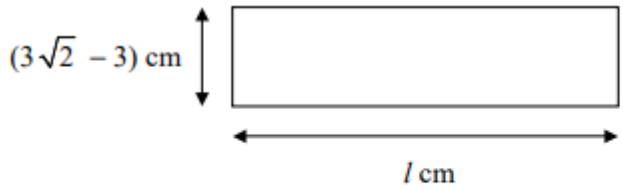
$$3x = \sqrt{5}(x+2),$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.

4. Express each of the following as simply as possible with a rational denominator.

a $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$ **b** $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ **c** $\frac{1+\sqrt{10}}{\sqrt{10}-3}$ **d** $\frac{3-\sqrt{2}}{4+3\sqrt{2}}$

5.



The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm.

Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

Exam Questions *(AQA Questions)*

1. **Jan 05 Q5**

(a) Simplify $(\sqrt{12}+2)(\sqrt{12}-2)$. *(2 marks)*

(b) Express $\sqrt{12}$ in the form $m\sqrt{3}$, where m is an integer. *(1 mark)*

(c) Express $\dfrac{\sqrt{12}+2}{\sqrt{12}-2}$ in the form $a+b\sqrt{3}$, where a and b are integers. *(4 marks)*

2. **June 05 Q5**

Express each of the following in the form $m+n\sqrt{3}$, where m and n are integers:

(a) $(\sqrt{3}+1)^2$; *(2 marks)*

(b) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$. *(3 marks)*

3. **Jan 06 Q1**

(a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

4. **June 06 Q4**

(a) Express $(4\sqrt{5}-1)(\sqrt{5}+3)$ in the form $p+q\sqrt{5}$, where p and q are integers. *(3 marks)*

(b) Show that $\dfrac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}$ is an integer and find its value. *(3 marks)*

5. **Jan 07 Q3**

(a) Express $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $p\sqrt{5}+q$, where p and q are integers. *(4 marks)*

(b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form. *(3 marks)*

6. **June 07 Q7**

(a) Express $\dfrac{\sqrt{63}}{3}+\dfrac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. *(3 marks)*

(b) Express $\dfrac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7}+q$, where p and q are integers. *(4 marks)*

Answers – Basic Skills

1.	**a** $=7$	**b** $=11$	**c** $=\frac{1}{3}$	
	d $=\frac{2}{5}$	**e** $=0.1$	**f** $=0.3$	
2.	**a** $=7$	**b** $=20$	**c** $=27$	**d** $=36$

3.

a $=\sqrt{4}\times\sqrt{3}=2\sqrt{3}$ **b** $=\sqrt{4}\times\sqrt{7}=2\sqrt{7}$ **c** $=\sqrt{16}\times\sqrt{5}=4\sqrt{5}$

d $=\sqrt{9}\times\sqrt{3}=3\sqrt{3}$ **e** $=\sqrt{4}\times\sqrt{6}=2\sqrt{6}$ **f** $=\sqrt{64}\times\sqrt{2}=8\sqrt{2}$

4.

a $=3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$ **b** $=4\sqrt{3}-3\sqrt{3}=\sqrt{3}$ **c** $=4\sqrt{2}+6\sqrt{2}=10\sqrt{2}$

5.

a $=\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{1}{5}\sqrt{5}$ **b** $=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2}{3}\sqrt{3}$ **c** $=\frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{1}{4}\sqrt{2}$

d $=\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=2\sqrt{7}$ **e** $=\frac{3\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\sqrt{6}$ **f** $=\frac{\sqrt{5}}{\sqrt{3}\sqrt{5}}=\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{1}{3}\sqrt{3}$

6.

a $=\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1$

b $=\frac{4}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{4(\sqrt{3}+1)}{3-1}=2(\sqrt{3}+1)$

c $=\frac{1}{\sqrt{6}-2}\times\frac{\sqrt{6}+2}{\sqrt{6}+2}=\frac{\sqrt{6}+2}{6-4}=\frac{1}{2}(\sqrt{6}+2)$ or $\frac{1}{2}\sqrt{6}+1$

d $=\frac{3}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{3(2-\sqrt{3})}{4-3}=3(2-\sqrt{3})$

Answers – Advanced Skills

1.

a $=10+3\sqrt{5}+2\sqrt{5}+3$
$=13+5\sqrt{5}$

b $=4\sqrt{2}-3-8+3\sqrt{2}$
$=7\sqrt{2}-11$

c $=28+12\sqrt{7}+9$
$=37+12\sqrt{7}$

2.

a $=2\sqrt{2}+\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$=2\sqrt{2}+3\sqrt{2}$
$=5\sqrt{2}$

b $=4\sqrt{3}-\frac{10}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$
$=4\sqrt{3}-\frac{10}{3}\sqrt{3}$
$=\frac{2}{3}\sqrt{3}$

c $=\frac{6-2\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$=\frac{6\sqrt{2}-4}{2}$
$=3\sqrt{2}-2$

d $=\frac{3\sqrt{5}-5}{2\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$
$=\frac{15-5\sqrt{5}}{10}$
$=\frac{1}{2}(3-\sqrt{5})$

e $=\frac{1}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}+\frac{1}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$=\frac{1}{6}\sqrt{2}+\frac{1}{8}\sqrt{2}$
$=\frac{7}{24}\sqrt{2}$

f $=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\frac{\sqrt{2}\sqrt{3}}{6\sqrt{2}}$
$=\frac{2}{3}\sqrt{3}-\frac{1}{6}\sqrt{3}$
$=\frac{1}{2}\sqrt{3}$

3.

$3x=\sqrt{5}x+2\sqrt{5}$
$x(3-\sqrt{5})=2\sqrt{5}$
$x=\frac{2\sqrt{5}}{3-\sqrt{5}}=\frac{2\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{2\sqrt{5}(3+\sqrt{5})}{9-5}$
$x=\frac{6\sqrt{5}+10}{4}=\frac{5}{2}+\frac{3}{2}\sqrt{5}$

4.

a $=\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}\times\frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}=\frac{\sqrt{2}(\sqrt{2}-\sqrt{6})}{2-6}=-\frac{1}{4}(2-2\sqrt{3})=\frac{1}{2}(\sqrt{3}-1)$

b $=\frac{1+\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{(1+\sqrt{3})(2-\sqrt{3})}{4-3}=2-\sqrt{3}+2\sqrt{3}-3=\sqrt{3}-1$

c $=\frac{1+\sqrt{10}}{\sqrt{10}-3}\times\frac{\sqrt{10}+3}{\sqrt{10}+3}=\frac{(1+\sqrt{10})(\sqrt{10}+3)}{10-9}=\sqrt{10}+3+10+3\sqrt{10}=13+4\sqrt{10}$

d $=\frac{3-\sqrt{2}}{4+3\sqrt{2}}\times\frac{4-3\sqrt{2}}{4-3\sqrt{2}}=\frac{(3-\sqrt{2})(4-3\sqrt{2})}{16-18}=\frac{12-9\sqrt{2}-4\sqrt{2}+6}{-2}=\frac{1}{2}(13\sqrt{2}-18)$ or $\frac{13}{2}\sqrt{2}-9$

5.

$l=\frac{6}{3\sqrt{2}-3}=\frac{6}{3\sqrt{2}-3}\times\frac{3\sqrt{2}+3}{3\sqrt{2}+3}=\frac{6(3\sqrt{2}+3)}{18-9}$
$l=\frac{18(\sqrt{2}+1)}{9}=2\sqrt{2}+2$

Exam Questions Solutions - Surds

1. Jan 05 Q5

(a) $(\sqrt{12}+2)(\sqrt{12}-2)$ (M1)

$= 12 - 2\sqrt{12} + 2\sqrt{12} - 4$

$= 8$ (A1)

(b) $\sqrt{12} = \sqrt{4}\sqrt{3}$

$= 2\sqrt{3}$ (B1)

(c) $\frac{(\sqrt{12}+2)(\sqrt{12}+2)}{(\sqrt{12}-2)(\sqrt{12}+2)}$ (M1)

$= \frac{12+2\sqrt{12}+2\sqrt{12}+4}{8}$ (A1)

$= \frac{16+4\sqrt{12}}{8}$

$= \frac{16+8\sqrt{3}}{8}$ (A1)

$= 2+\sqrt{3}$ (A1)

2. June 05 Q5

(a) $(\sqrt{3}+1)^2$

$= (\sqrt{3}+1)(\sqrt{3}+1)$ (M1)

$= 3+\sqrt{3}+\sqrt{3}+1$

$= 4+2\sqrt{3}$ (A1)

(b) $\frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ (M1)

$= \frac{4+2\sqrt{3}}{3+\sqrt{3}-\sqrt{3}-1}$ (A1)

$= \frac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3}$ (A1)

3. Jan 06 Q1

(a) $(\sqrt{5}+2)(\sqrt{5}-2)$

$= 5 - 2\sqrt{5} + 2\sqrt{5} - 4$ (M1)

$= 1$ (A1)

(b) $\sqrt{8} + \sqrt{18}$

$= \sqrt{4}\sqrt{2} + \sqrt{9}\sqrt{2}$ (M1)

$= 2\sqrt{2} + 3\sqrt{2}$

$= 5\sqrt{2}$ (A1)

4. June 06 Q4

(a) $(4\sqrt{5}-1)(\sqrt{5}+3)$

$= 20 + 12\sqrt{5} - \sqrt{5} - 3$ (M1)(A1)

$= 17 + 11\sqrt{5}$ (A1)

(b) $\frac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}$

$= \frac{5\sqrt{3}-3\sqrt{3}}{\sqrt{3}}$ (M1)

$= \frac{2\sqrt{3}}{\sqrt{3}}$ (M1)

$= 2$ (A1)

5. Jan 07 Q3

(a) $\frac{(\sqrt{5}+3)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$ (M1)

$= \frac{5+2\sqrt{5}+3\sqrt{5}+6}{5-4}$ (A1) (A1)

$= 11 + 5\sqrt{5}$ (A1)

(b)(i) $\sqrt{45} = \sqrt{9}\sqrt{5}$ (B1)

$= 3\sqrt{5}$

(ii) $x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$

$2x\sqrt{5} = 7\sqrt{5} - 3\sqrt{5}$ (M1)

$2x\sqrt{5} = 4\sqrt{5}$ (M1)

$2x = 4$

$x = 2$ (A1)

6. June 07 Q7

(a) $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$

(M1)

$= \frac{3\sqrt{7}}{3} + \frac{14}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ (M1)

$= \frac{3\sqrt{7}}{3} + \frac{14\sqrt{7}}{7}$

$= \sqrt{7} + 2\sqrt{7}$

$= 3\sqrt{7}$ (A1)

(b) $\frac{(\sqrt{7}+1)(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$ (M1)

$= \frac{7+2\sqrt{7}+\sqrt{7}+2}{7-4}$ (A1) (A1)

$= \frac{9+3\sqrt{7}}{3}$

$= 3 + \sqrt{7}$ (A1)

Topic: B3 Quadratics **Basic Skills**

https://youtu.be/Pziws8ojnlk https://youtu.be/sn_joGVj15w

https://youtu.be/kk7p6hjn7hQ https://youtu.be/toIqbX_NXHo

newcollege Doncaster
Mathematics Department

1. Using factorisation, solve each equation.

a $x^2 - 4x + 3 = 0$ **b** $x^2 + 6x + 8 = 0$ **c** $x^2 + 4x - 5 = 0$ **d** $x^2 - 7x = 8$

e $x^2 - 25 = 0$ **f** $x(x-1) = 42$ **g** $x^2 = 3x$ **h** $27 + 12x + x^2 = 0$

i $60 - 4x - x^2 = 0$ **j** $5x + 14 = x^2$ **k** $2x^2 - 3x + 1 = 0$ **l** $x(x-1) = 6(x-2)$

2.

a $x - 5 + \frac{4}{x} = 0$ **b** $x - \frac{10}{x} = 3$ **c** $2x^3 - x^2 - 3x = 0$ **d** $x^2(10 - x^2) = 9$

e $\frac{5}{x^2} + \frac{4}{x} - 1 = 0$ **f** $\frac{x-6}{x-4} = x$ **g** $x + 5 = \frac{3}{x+3}$ **h** $x^2 - \frac{4}{x^2} = 3$

3. Sketch each curve showing the coordinates of any points of intersection with the coordinate axes.

a $y = x^2 - 3x + 2$ **b** $y = x^2 + 5x + 6$ **c** $y = x^2 - 9$

d $y = x^2 - 2x$ **e** $y = x^2 - 10x + 25$ **f** $y = 2x^2 - 14x + 20$

4. Use the quadratic formula to solve each equation, giving your answers as simply as possible in terms of surds where appropriate.

a $x^2 + 4x + 1 = 0$ **b** $4 + 8t - t^2 = 0$ **c** $y^2 - 20y + 91 = 0$ **d** $r^2 + 2r - 7 = 0$

e $6 + 18a + a^2 = 0$ **f** $m(m-5) = 5$ **g** $x^2 + 11x + 27 = 0$ **h** $2u^2 + 6u + 3 = 0$

i $5 - y - y^2 = 0$ **j** $2x^2 - 3x = 2$ **k** $3p^2 + 7p + 1 = 0$ **l** $t^2 - 14t = 14$

5. Express in the form $(x + a)^2 + b$

a $x^2 + 2x + 4$ **b** $x^2 - 2x + 4$ **c** $x^2 - 4x + 1$ **d** $x^2 + 6x$

e $x^2 + 4x + 8$ **f** $x^2 - 8x - 5$ **g** $x^2 + 12x + 30$ **h** $x^2 - 10x + 25$

i $x^2 + 6x - 9$ **j** $18 - 4x + x^2$ **k** $x^2 + 3x + 3$ **l** $x^2 + x - 1$

6. Express in the form $a(x + b)^2 + c$

a $2x^2 + 4x + 3$ **b** $2x^2 - 8x - 7$ **c** $3 - 6x + 3x^2$ **d** $4x^2 + 24x + 11$

e $-x^2 - 2x - 5$ **f** $1 + 10x - x^2$ **g** $2x^2 + 2x - 1$ **h** $3x^2 - 9x + 5$

i $3x^2 - 24x + 48$ **j** $3x^2 - 15x$ **k** $70 + 40x + 5x^2$ **l** $2x^2 + 5x + 2$

m $4x^2 + 6x - 7$ **n** $-2x^2 + 4x - 1$ **o** $4 - 2x - 3x^2$ **p** $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$

7. Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate.

a $y^2 - 4y + 2 = 0$ **b** $p^2 + 2p - 2 = 0$ **c** $x^2 - 6x + 4 = 0$ **d** $7 + 10r + r^2 = 0$

e $x^2 - 2x = 11$ **f** $a^2 - 12a - 18 = 0$ **g** $m^2 - 3m + 1 = 0$ **h** $9 - 7t + t^2 = 0$

i $u^2 + 7u = 44$ **j** $2y^2 - 4y + 1 = 0$ **k** $3p^2 + 18p = -23$ **l** $2x^2 + 12x = 9$

8. Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the y-axis.

a $y = x^2 - 4x + 3$ **b** $y = x^2 + 2x - 24$ **c** $y = x^2 - 2x + 5$

d $y = 30 + 8x + x^2$ **e** $y = x^2 + 2x + 1$ **f** $y = 8 + 2x - x^2$

g $y = -x^2 + 8x - 7$ **h** $y = -x^2 - 4x - 7$ **i** $y = x^2 - 5x + 4$

j $y = x^2 + 3x + 3$ **k** $y = 3 + 8x + 4x^2$ **l** $y = -2x^2 + 8x - 15$

...include points of intesection with the x-axis

9.	By letting $y = 2^x$, or otherwise, solve the equation $2^{2x} - 10(2^x) + 16 = 0.$

Advanced Skills

1.	Solve the equations **a** $x - \frac{5}{x} = 4$ **b** $\frac{9}{5-x} - 1 = 2x$
2.	Find in the form $k\sqrt{2}$ the solutions of the equation $2x^2 + 5\sqrt{2}\,x - 6 = 0.$
3.	**a** Express $x^2 - 4\sqrt{2}\,x + 5$ in the form $a(x+b)^2 + c$. **b** Write down an equation of the line of symmetry of the curve $y = x^2 + 4\sqrt{2}\,x + 5$.
4.	$f(x) \equiv x^2 + 2kx - 3.$ By completing the square, find the roots of the equation $f(x) = 0$ in terms of the constant k.
5.	Labelling the coordinates of any points of intersection with the coordinate axes, sketch the curves **a** $y = (x+1)(x-p)$ where $p > 0$, **b** $y = (x+q)^2$ where $q < 0$.
6.	$f(x) \equiv 2x^2 - 6x + 5.$ **a** Find the values of A, B and C such that $f(x) \equiv A(x+B)^2 + C.$ **b** Hence deduce the minimum value of $f(x)$.
7.	**a** Given that $t = x^{\frac{1}{3}}$ express $x^{\frac{2}{3}}$ in terms of t. **b** Hence, or otherwise, solve the equation $2x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0.$
8.	**a** Given that $y = 3^x$ express 3^{2x+2} in terms of y. **b** Hence, or otherwise, solve the equation $3^{2x+2} - 10(3^x) + 1 = 0.$

Exam Questions *(AQA C1 Questions)*

1.	**Jan 2011 Q7** **(a) (i)** Express $4 - 10x - x^2$ in the form $p - (x+q)^2$. *(2 marks)* **(ii)** Hence write down the equation of the line of symmetry of the curve with equation $y = 4 - 10x - x^2$. *(1 mark)*

2. **June 11 Q4**

(a) Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where p and q are rational numbers. *(3 marks)*

(b) A curve has equation $y = x^2 + 5x + 7$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) State the equation of the line of symmetry of the curve. *(1 mark)*

(iii) Sketch the curve, stating the value of the intercept on the y-axis. *(3 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. *(3 marks)*

3. **Jan 12 Q2**

(a) Factorise $x^2 - 4x - 12$. *(1 mark)*

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes. *(4 marks)*

(c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where p and q are positive integers. *(2 marks)*

(ii) Hence find the minimum value of $x^2 - 4x - 12$. *(1 mark)*

(d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

Find an equation of the new curve. You need not simplify your answer. *(2 marks)*

4. **June 12 Q5**

(a) (i) Express $x^2 - 3x + 5$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*

5. **Jan 13 Q4**

(a) (i) Express $x^2 - 6x + 11$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions. *(2 marks)*

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) Sketch the curve, indicating the value of y where the curve crosses the y-axis. *(3 marks)*

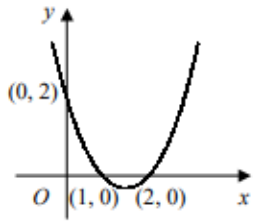
(iii) Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$. *(3 marks)*

6.	**June 13 Q5** **(a) (i)** Express $2x^2 + 6x + 5$ in the form $2(x+p)^2 + q$, where p and q are rational numbers. *(2 marks)* **(ii)** Hence write down the minimum value of $2x^2 + 6x + 5$. *(1 mark)*

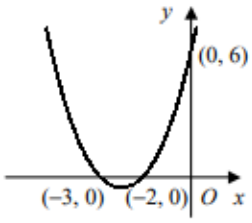
Answers – Basic Skills

1.	**a** $(x-1)(x-3)=0$, $x=1$ or 3 **b** $(x+4)(x+2)=0$, $x=-4$ or -2 **c** $(x+5)(x-1)=0$, $x=-5$ or 1 **d** $x^2-7x-8=0$, $(x+1)(x-8)=0$, $x=-1$ or 8 **e** $(x+5)(x-5)=0$, $x=-5$ or 5 **f** $x^2-x-42=0$, $(x+6)(x-7)=0$, $x=-6$ or 7 **g** $x^2-3x=0$, $x(x-3)=0$, $x=0$ or 3 **h** $(x+9)(x+3)=0$, $x=-9$ or -3 **i** $x^2+4x-60=0$, $(x+10)(x-6)=0$, $x=-10$ or 6 **j** $x^2-5x-14=0$, $(x+2)(x-7)=0$, $x=-2$ or 7 **k** $(2x-1)(x-1)=0$, $x=\frac{1}{2}$ or 1 **l** $x^2-x=6x-12$, $x^2-7x+12=0$, $(x-3)(x-4)=0$, $x=3$ or 4
2.	**a** $x^2-5x+4=0$, $(x-1)(x-4)=0$, $x=1$ or 4 **b** $x^2-10=3x$, $x^2-3x-10=0$, $(x+2)(x-5)=0$, $x=-2$ or 5 **c** $x(2x^2-x-3)=0$, $x(2x-3)(x+1)=0$, $x=-1, 0$ or $\frac{3}{2}$ **d** $10x^2-x^4=9$, $x^4-10x^2+9=0$, $(x^2-1)(x^2-9)=0$, $x^2=1$ or 9, $x=\pm 1$ or ± 3 **e** $5+4x-x^2=0$, $x^2-4x-5=0$, $(x+1)(x-5)=0$, $x=-1$ or 5 **f** $x-6=x(x-4)$, $x-6=x^2-4x$, $x^2-5x+6=0$, $(x-2)(x-3)=0$, $x=2$ or 3 **g** $(x+5)(x+3)=3$, $x^2+8x+15=3$, $x^2+8x+12=0$, $(x+6)(x+2)=0$, $x=-6$ or -2 **h** $x^4-4=3x^2$, $x^4-3x^2-4=0$, $(x^2+1)(x^2-4)=0$, $x^2=-1$ (no sol's) or 4, $x=\pm 2$
3.	

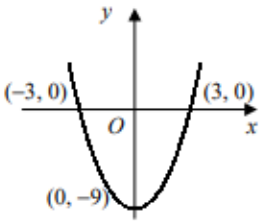
a $x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$
$x = 1$ or 2



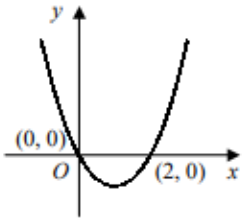
b $x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ or -2



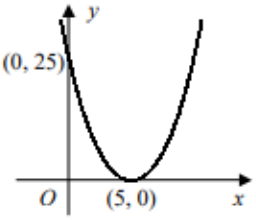
c $x^2 - 9 = 0$
$(x + 3)(x - 3) = 0$
$x = -3$ or 3



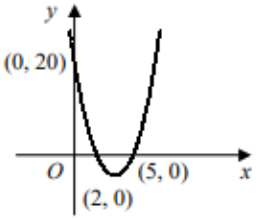
d $x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0$ or 2



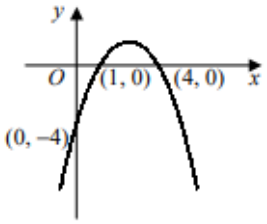
e $x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x = 5$



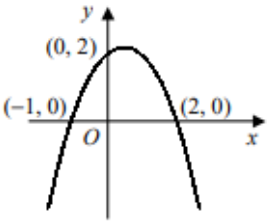
f $2x^2 - 14x + 20 = 0$
$2(x - 2)(x - 5) = 0$
$x = 2$ or 5



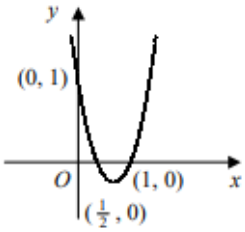
g $-x^2 + 5x - 4 = 0$
$x^2 - 5x + 4 = 0$
$(x - 1)(x - 4) = 0$
$x = 1$ or 4



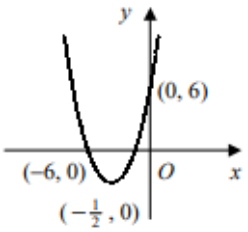
h $2 + x - x^2 = 0$
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2



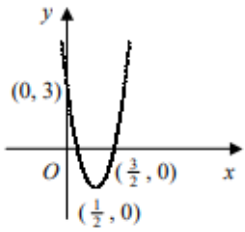
i $2x^2 - 3x + 1 = 0$
$(2x - 1)(x - 1) = 0$
$x = \frac{1}{2}$ or 1



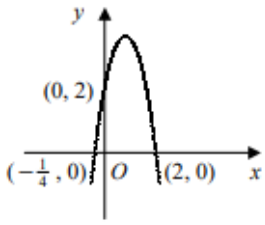
j $2x^2 + 13x + 6 = 0$
$(2x + 1)(x + 6) = 0$
$x = -6$ or $-\frac{1}{2}$



k $3 - 8x + 4x^2 = 0$
$(2x - 1)(2x - 3) = 0$
$x = \frac{1}{2}$ or $\frac{3}{2}$



l $2 + 7x - 4x^2 = 0$
$4x^2 - 7x - 2 = 0$
$(4x + 1)(x - 2) = 0$
$x = -\frac{1}{4}$ or 2



4.

a $x = \dfrac{-4 \pm \sqrt{16-4}}{2}$

$x = \dfrac{-4 \pm 2\sqrt{3}}{2}$

$x = -2 \pm \sqrt{3}$

b $t = \dfrac{-8 \pm \sqrt{64+16}}{-2}$

$t = \dfrac{-8 \pm 4\sqrt{5}}{-2}$

$t = 4 \pm 2\sqrt{5}$

c $y = \dfrac{20 \pm \sqrt{400-364}}{2}$

$y = \dfrac{20 \pm 6}{2}$

$y = 7$ or 13

d $r = \dfrac{-2 \pm \sqrt{4+28}}{2}$

$r = \dfrac{-2 \pm 4\sqrt{2}}{2}$

$r = -1 \pm 2\sqrt{2}$

e $a = \dfrac{-18 \pm \sqrt{324-24}}{2}$

$a = \dfrac{-18 \pm 10\sqrt{3}}{2}$

$a = -9 \pm 5\sqrt{3}$

f $m^2 - 5m - 5 = 0$

$m = \dfrac{5 \pm \sqrt{25+20}}{2}$

$m = \frac{1}{2}(5 \pm 3\sqrt{5})$

g $x = \dfrac{-11 \pm \sqrt{121-108}}{2}$

$x = \frac{1}{2}(-11 \pm \sqrt{13})$

h $u = \dfrac{-6 \pm \sqrt{36-24}}{4}$

$u = \dfrac{-6 \pm 2\sqrt{3}}{4}$

$u = \frac{1}{2}(-3 \pm \sqrt{3})$

i $y = \dfrac{1 \pm \sqrt{1+20}}{-2}$

$y = -\frac{1}{2}(1 \pm \sqrt{21})$

j $2x^2 - 3x - 2 = 0$

$x = \dfrac{3 \pm \sqrt{9+16}}{4}$

$x = \dfrac{3 \pm 5}{4}$

$x = -\frac{1}{2}$ or 2

k $p = \dfrac{-7 \pm \sqrt{49-12}}{6}$

$p = \frac{1}{6}(-7 \pm \sqrt{37})$

l $t^2 - 14t - 14 = 0$

$t = \dfrac{14 \pm \sqrt{196+56}}{2}$

$t = \dfrac{14 \pm 6\sqrt{7}}{2}$

$t = 7 \pm 3\sqrt{7}$

5.

a $= (x+1)^2 - 1 + 4$

$= (x+1)^2 + 3$

b $= (x-1)^2 - 1 + 4$

$= (x-1)^2 + 3$

c $= (x-2)^2 - 4 + 1$

$= (x-2)^2 - 3$

d $= (x+3)^2 - 9$

e $= (x+2)^2 - 4 + 8$

$= (x+2)^2 + 4$

f $= (x-4)^2 - 16 - 5$

$= (x-4)^2 - 21$

g $= (x+6)^2 - 36 + 30$

$= (x+6)^2 - 6$

h $= (x-5)^2 - 25 + 25$

$= (x-5)^2$

i $= (x+3)^2 - 9 - 9$

$= (x+3)^2 - 18$

j $= (x-2)^2 - 4 + 18$

$= (x-2)^2 + 14$

k $= (x+\frac{3}{2})^2 - \frac{9}{4} + 3$

$= (x+\frac{3}{2})^2 + \frac{3}{4}$

l $= (x+\frac{1}{2})^2 - \frac{1}{4} - 1$

$= (x+\frac{1}{2})^2 - \frac{5}{4}$

6.

a $= 2[x^2 + 2x] + 3$

$= 2[(x+1)^2 - 1] + 3$

$= 2(x+1)^2 + 1$

b $= 2[x^2 - 4x] - 7$

$= 2[(x-2)^2 - 4] - 7$

$= 2(x-2)^2 - 15$

c $= 3[x^2 - 2x] + 3$

$= 3[(x-1)^2 - 1] + 3$

$= 3(x-1)^2$

d $= 4[x^2 + 6x] + 11$

$= 4[(x+3)^2 - 9] + 11$

$= 4(x+3)^2 - 25$

e $= -[x^2 + 2x] - 5$

$= -[(x+1)^2 - 1] - 5$

$= -(x+1)^2 - 4$

f $= -[x^2 - 10x] + 1$

$= -[(x-5)^2 - 25] + 1$

$= -(x-5)^2 + 26$

g $= 2[x^2 + x] - 1$

$= 2[(x+\frac{1}{2})^2 - \frac{1}{4}] - 1$

$= 2(x+\frac{1}{2})^2 - \frac{3}{2}$

h $= 3[x^2 - 3x] + 5$

$= 3[(x-\frac{3}{2})^2 - \frac{9}{4}] + 5$

$= 3(x-\frac{3}{2})^2 - \frac{7}{4}$

i $= 3[x^2 - 8x] + 48$

$= 3[(x-4)^2 - 16] + 48$

$= 3(x-4)^2$

j $= 3[x^2 - 5x]$

$= 3[(x-\frac{5}{2})^2 - \frac{25}{4}]$

$= 3(x-\frac{5}{2})^2 - \frac{75}{4}$

k $= 5[x^2 + 8x] + 70$

$= 5[(x+4)^2 - 16] + 70$

$= 5(x+4)^2 - 10$

l $= 2[x^2 + \frac{5}{2}x] + 2$

$= 2[(x+\frac{5}{4})^2 - \frac{25}{16}] + 2$

$= 2(x+\frac{5}{4})^2 - \frac{9}{8}$

m $= 4[x^2 + \frac{3}{2}x] - 7$

$= 4[(x+\frac{3}{4})^2 - \frac{9}{16}] - 7$

$= 4(x+\frac{3}{4})^2 - \frac{37}{4}$

n $= -2[x^2 - 2x] - 1$

$= -2[(x-1)^2 - 1] - 1$

$= -2(x-1)^2 + 1$

o $= -3[x^2 + \frac{2}{3}x] + 4$

$= -3[(x+\frac{1}{3})^2 - \frac{1}{9}] + 4$

$= -3(x+\frac{1}{3})^2 + \frac{13}{3}$

p $= \frac{1}{3}[x^2 + \frac{3}{2}x] - \frac{1}{4}$

$= \frac{1}{3}[(x+\frac{3}{4})^2 - \frac{9}{16}] - \frac{1}{4}$

$= \frac{1}{3}(x+\frac{3}{4})^2 - \frac{7}{16}$

7.

a $(y-2)^2-4+2=0$
$(y-2)^2=2$
$y-2=\pm\sqrt{2}$
$y=2\pm\sqrt{2}$

b $(p+1)^2-1-2=0$
$(p+1)^2=3$
$p+1=\pm\sqrt{3}$
$p=-1\pm\sqrt{3}$

c $(x-3)^2-9+4=0$
$(x-3)^2=5$
$x-3=\pm\sqrt{5}$
$x=3\pm\sqrt{5}$

d $(r+5)^2-25+7=0$
$(r+5)^2=18$
$r+5=\pm\sqrt{18}=\pm3\sqrt{2}$
$r=-5\pm3\sqrt{2}$

e $(x-1)^2-1=11$
$(x-1)^2=12$
$x-1=\pm\sqrt{12}=\pm2\sqrt{3}$
$x=1\pm2\sqrt{3}$

f $(a-6)^2-36-18=0$
$(a-6)^2=54$
$a-6=\pm\sqrt{54}=\pm3\sqrt{6}$
$a=6\pm3\sqrt{6}$

g $(m-\frac{3}{2})^2-\frac{9}{4}+1=0$
$(m-\frac{3}{2})^2=\frac{5}{4}$
$m-\frac{3}{2}=\pm\frac{\sqrt{5}}{2}$
$m=\frac{1}{2}(3\pm\sqrt{5})$

h $(t-\frac{7}{2})^2-\frac{49}{4}+9=0$
$(t-\frac{7}{2})^2=\frac{13}{4}$
$t-\frac{7}{2}=\pm\frac{\sqrt{13}}{2}$
$t=\frac{1}{2}(7\pm\sqrt{13})$

i $(u+\frac{7}{2})^2-\frac{49}{4}=44$
$(u+\frac{7}{2})^2=\frac{225}{4}$
$u+\frac{7}{2}=\pm\frac{15}{2}$
$u=-\frac{7}{2}\pm\frac{15}{2}$
$u=-11$ or 4

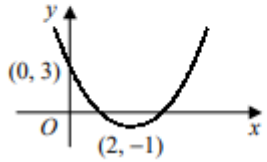
j $y^2-2y+\frac{1}{2}=0$
$(y-1)^2-1+\frac{1}{2}=0$
$(y-1)^2=\frac{1}{2}$
$y-1=\pm\frac{1}{\sqrt{2}}=\pm\frac{1}{2}\sqrt{2}$
$y=1\pm\frac{1}{2}\sqrt{2}$

k $p^2+6p=-\frac{23}{3}$
$(p+3)^2-9=-\frac{23}{3}$
$(p+3)^2=\frac{4}{3}$
$p+3=\pm\frac{2}{\sqrt{3}}=\pm\frac{2}{3}\sqrt{3}$
$p=-3\pm\frac{2}{3}\sqrt{3}$

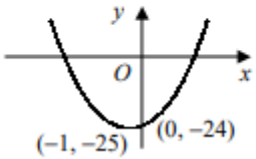
l $x^2+6x=\frac{9}{2}$
$(x+3)^2-9=\frac{9}{2}$
$(x+3)^2=\frac{27}{2}$
$x+3=\pm\sqrt{\frac{27}{2}}=\pm\frac{3}{2}\sqrt{6}$
$x=-3\pm\frac{3}{2}\sqrt{6}$

8.

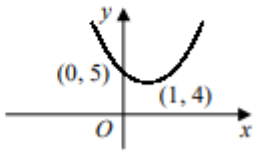
a $y=(x-2)^2-4+3$
$y=(x-2)^2-1$
minimum $(2,-1)$



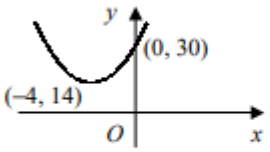
b $y=(x+1)^2-1-24$
$y=(x+1)^2-25$
minimum $(-1,-25)$



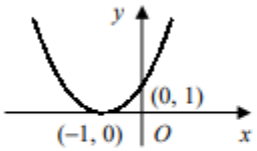
c $y=(x-1)^2-1+5$
$y=(x-1)^2+4$
minimum $(1,4)$



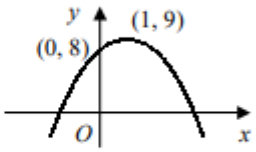
d $y=(x+4)^2-16+30$
$y=(x+4)^2+14$
minimum $(-4,14)$



e $y=(x+1)^2-1+1$
$y=(x+1)^2$
minimum $(-1,0)$



f $y=-[x^2-2x]+8$
$y=-[(x-1)^2-1]+8$
$y=-(x-1)^2+9$
maximum $(1,9)$

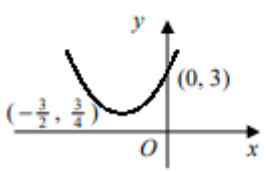


g $y=-[x^2-8x]-7$
$y=-[(x-4)^2-16]-7$
$y=-(x-4)^2+9$
maximum $(4,9)$

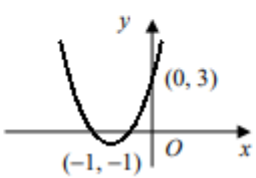
h $y=-[x^2+4x]-7$
$y=-[(x+2)^2-4]-7$
$y=-(x+2)^2-3$
maximum $(-2,-3)$

i $y=(x-\frac{5}{2})^2-\frac{25}{4}+4$
$y=(x-\frac{5}{2})^2-\frac{9}{4}$
minimum $(\frac{5}{2},-\frac{9}{4})$

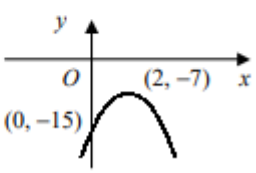
j $y=(x+\frac{3}{2})^2-\frac{9}{4}+3$
$y=(x+\frac{3}{2})^2+\frac{3}{4}$
minimum $(-\frac{3}{2},\frac{3}{4})$



k $y=4[x^2+2x]+3$
$y=4[(x+1)^2-1]+3$
$y=4(x+1)^2-1$
minimum $(-1,-1)$



l $y=-2[x^2-4x]-15$
$y=-2[(x-2)^2-4]-15$
$y=-2(x-2)^2-7$
maximum $(2,-7)$



9.

$y^2 - 10y + 16 = 0$
$(y - 2)(y - 8) = 0$
$y = 2^x = 2$ or 8
$x = 1$ or 3

Answers – Advanced Skills

1.

a $x^2 - 5 = 4x$
$x^2 - 4x - 5 = 0$
$(x + 1)(x - 5) = 0$
$x = -1$ or 5

b $9 - (5 - x) = 2x(5 - x)$
$2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$x = \frac{1}{2}$ or 4

2.

$x = \frac{-5\sqrt{2} \pm \sqrt{50 + 48}}{4}$
$= \frac{-5\sqrt{2} \pm \sqrt{98}}{4}$
$= \frac{-5\sqrt{2} \pm 7\sqrt{2}}{4}$
$= -3\sqrt{2}$ or $\frac{1}{2}\sqrt{2}$

3.

a $= (x - 2\sqrt{2})^2 - 8 + 5$
$= (x - 2\sqrt{2})^2 - 3$

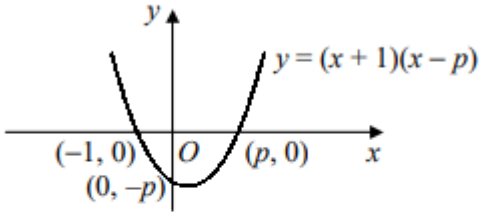
b $x = 2\sqrt{2}$

4.

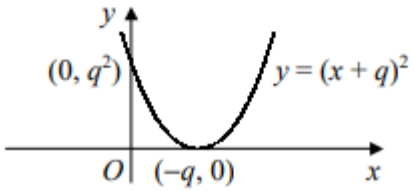
$x^2 + 2kx - 3 = 0$
$(x + k)^2 - k^2 - 3 = 0$
$(x + k)^2 = k^2 + 3$
$x + k = \pm\sqrt{k^2 + 3}$
$x = -k \pm \sqrt{k^2 + 3}$

5.

a $x = 0 \Rightarrow y = -p$
$y = 0 \Rightarrow x = -1$ or p



b $x = 0 \Rightarrow y = q^2$
$y = 0 \Rightarrow x = -q \quad [-q > 0]$



6.

a $f(x) = 2[x^2 - 3x] + 5$
$= 2[(x - \frac{3}{2})^2 - \frac{9}{4}] + 5$
$= 2(x - \frac{3}{2})^2 + \frac{1}{2}$
$\therefore A = 2, B = -\frac{3}{2}, C = \frac{1}{2}$

b minimum value of $f(x) = \frac{1}{2}$

7.

a $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = t^2$

b let $t = x^{\frac{1}{3}} \Rightarrow 2t^2 + t - 6 = 0$
$(2t - 3)(t + 2) = 0$
$t = -2$ or $\frac{3}{2}$
but $x = t^3 \quad \therefore \quad x = -8$ or $\frac{27}{8}$

8.

a $3^{2x+2} = 3^2(3^x)^2 = 9y^2$

b $9y^2 - 10y + 1 = 0$

$(9y - 1)(y - 1) = 0$

$y = 3^x = \frac{1}{9}, 1$

$\therefore x = -2, 0$

Quadratics Exam Questions Solutions

1. Jan 2011 Q7

(a)(i) $4 - 10x - x^2$

$\equiv -(x^2 + 10x - 4)$

$\equiv -((x+5)^2 - 25 - 4)$ (M1)

$\equiv -((x+5)^2 - 29)$

$\equiv 29 - (x+5)^2$ (A1)

(ii) line of symmetry $x = -5$ (B1FT)

2. June 11 Q4

(a) $x^2 + 5x + 7 \equiv$

$\equiv (x + 5/2)^2 - \frac{25}{4} + \frac{28}{4}$ (B1)(M1)

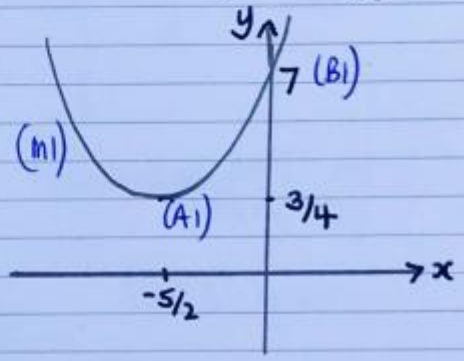
$\equiv (x + \frac{5}{2})^2 + \frac{3}{4}$ (A1)

(b) (i) when $x = -\frac{5}{2}$ $y = \frac{3}{4}$ (M1)

$\therefore$ vertex at $(-5/2, 3/4)$ (A1)

(ii) line of symmetry $x = -\frac{5}{2}$ (B1FT)

(iii)



(c) Translation $\begin{pmatrix} -5/2 \\ 3/4 \end{pmatrix}$ (M1)(A1)

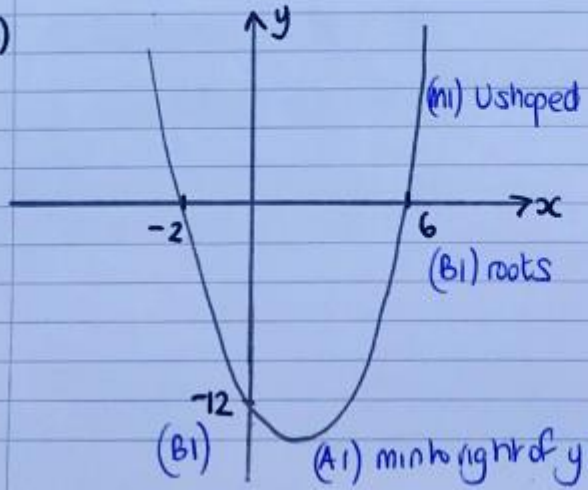
(E1)

3. Jan 12 Q2

(a) $x^2 - 4x - 12$

$\equiv (x - 6)(x + 2)$ (B1)

(b)



(c)(i) $x^2 - 4x - 12$

$\equiv (x-2)^2 - 4 - 12$ (M1)

$\equiv (x-2)^2 - 16$ (A1)

(ii) min value is -16 (B1FT)

(d) $y = (x+3)^2 - 4(x+3) - 12 + 2$ (M1)

$y = (x+3)^2 - 4(x+3) - 10$

or any equivalent form (A1)

ie $y = (x+1)^2 - 14$

4. June 12 Q5

(a)(i) $x^2 - 3x + 5$

$\equiv (x - 3/2)^2 - \frac{9}{4} + \frac{20}{4}$ (M1)

$\equiv (x - 3/2)^2 + \frac{11}{4}$ (A1)

(ii) line of symmetry is $x = 3/2$ (B1FT)

5. Jan 13 Q4

(a)(i) $x^2 - 6x + 11$

$\equiv (x-3)^2 - 9 + 11$ (M1)

$\equiv (x-3)^2 + 2$ (A1)

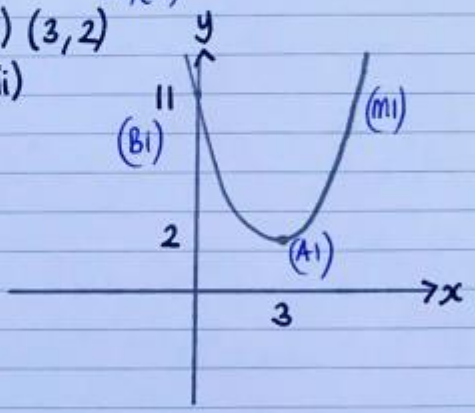
(ii) $(x-3)^2 + 2 = 0$

$(x-3)^2 = -2$ (M1)

can't take square root of negative number and get real solutions ∴ no real solutions (A1)

(b)(i) (3, 2) (M1)(A1)

(b)(ii)



(iii) Translation $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ (M1) (E1) (A1)

Gone backwards !

6. June 13 Q5

(a)(i) $2x^2 + 6x + 5$

$\equiv 2[x^2 + 3x] + 5$ (M1)

$\equiv 2[(x+3/2)^2 - 9/4] + 5$

$\equiv 2(x+3/2)^2 - 9/2 + 10/2$

$\equiv 2(x+3/2)^2 + 1/2$ (A1)

or $2x^2 + 6x + 5$

$\equiv 2[x^2 + 3x + 5/2]$

$\equiv 2[(x+3/2)^2 - 9/4 + 10/4]$

$\equiv 2[(x+3/2)^2 + 1/4]$

$\equiv 2(x+3/2)^2 + 1/2$

(ii) Min value is $y = 1/2$ (B1FT)

Topic: B4 Simultaneous Equations

Basic Skills

https://youtu.be/4SRtwS5unwE

newcollege Doncaster
Mathematics Department

1. Solve each pair of simultaneous equations.

a $y = 3x$
$y = 2x + 1$

b $y = x - 6$
$y = \frac{1}{2}x - 4$

c $y = 2x + 6$
$y = 3 - 4x$

d $x + y - 3 = 0$
$x + 2y + 1 = 0$

e $x + 2y + 11 = 0$
$2x - 3y + 1 = 0$

f $3x + 3y + 4 = 0$
$5x - 2y - 5 = 0$

2. Solve each pair of simultaneous equations.

a $x^2 - y + 3 = 0$
$x - y + 5 = 0$

b $2x^2 - y - 8x = 0$
$x + y + 3 = 0$

c $x^2 + y^2 = 25$
$2x - y = 5$

d $x^2 + 2xy + 15 = 0$
$2x - y + 10 = 0$

e $x^2 - 2xy - y^2 = 7$
$x + y = 1$

f $3x^2 - x - y^2 = 0$
$x + y - 1 = 0$

g $2x^2 + xy + y^2 = 22$
$x + y = 4$

h $x^2 - 4y - y^2 = 0$
$x - 2y = 0$

i $x^2 + xy = 4$
$3x + 2y = 6$

3. Find in each case the coordinates of the points where the line l intersects the circle C.

a $l : y = x - 4$ $C : x^2 + y^2 = 10$

b $l : 3x + y = 17$ $C : x^2 + y^2 - 4x - 2y - 15 = 0$

c $l : y = 2x + 2$ $C : 4x^2 + 4y^2 + 4x - 8y - 15 = 0$

4. Show that the line with equation $y = 2x + 1$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 8y + 27 = 0$ and find the coordinates of the point where they touch.

Advanced Skills

1. Solve the simultaneous equations

$$2x^2 - y^2 - 7 = 0$$
$$2x - 3y + 7 = 0$$

2. Solve each pair of simultaneous equations.

a $x - \frac{1}{y} - 4y = 0$
$x - 6y - 1 = 0$

b $xy = 6$
$x - y = 5$

c $\frac{3}{x} - 2y + 4 = 0$
$4x + y - 7 = 0$

3. Solve the simultaneous equations

$$3^{x-1} = 9^{2y}$$
$$8^{x-2} = 4^{1+y}$$

4. Solve the simultaneous equations

$$4^{2x} = 2^{y-1}$$
$$9^{4x} = 3^{y+1}$$

Exam Questions *(AQA C1 Questions)*

1. **Jan 011 Q7**

(b) The curve C has equation $y = 4 - 10x - x^2$ and the line L has equation $y = k(4x - 13)$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$x^2 + 2(2k + 5)x - (13k + 4) = 0 \qquad \textit{(1 mark)}$$

2. **Jan 13 Q8**

A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the curve and the line satisfies the equation

$$2x^2 - (2k + 1)x + 3k - 1 = 0 \qquad \textit{(1 mark)}$$

Answers – Basic Skills

1.

a $3x = 2x + 1$
$x = 1$
$\therefore\ x = 1, y = 3$

b $x - 6 = \frac{1}{2}x - 4$
$x = 4$
$\therefore\ x = 4, y = -2$

c $2x + 6 = 3 - 4x$
$x = -\frac{1}{2}$
$\therefore\ x = -\frac{1}{2}, y = 5$

d subtracting
$y + 4 = 0$
$y = -4$
$\therefore\ x = 7, y = -4$

e $2x + 4y + 22 = 0$
$2x - 3y + 1 = 0$
subtracting
$7y + 21 = 0$
$y = -3$
$\therefore\ x = -5, y = -3$

f $6x + 6y + 8 = 0$
$15x - 6y - 15 = 0$
adding
$21x - 7 = 0$
$x = \frac{1}{3}$
$\therefore\ x = \frac{1}{3}, y = -\frac{5}{3}$

2.

a subtracting
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2
$\therefore\ x = -1, y = 4$
or $x = 2, y = 7$

b adding
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or 3
$\therefore\ x = \frac{1}{2}, y = -\frac{7}{2}$
or $x = 3, y = -6$

c $y = 2x - 5$
sub
$x^2 + (2x - 5)^2 = 25$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ or 4
$\therefore\ x = 0, y = -5$
or $x = 4, y = 3$

d $y = 2x + 10$
sub.
$x^2 + 2x(2x + 10) + 15 = 0$
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3$ or -1
$\therefore\ x = -3, y = 4$
or $x = -1, y = 8$

e $y = 1 - x$
sub.
$x^2 - 2x(1 - x) - (1 - x)^2 = 7$
$x^2 = 4$
$x = \pm 2$
$\therefore\ x = -2, y = 3$
or $x = 2, y = -1$

f $y = 1 - x$
sub.
$3x^2 - x - (1 - x)^2 = 0$
$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
$x = -1$ or $\frac{1}{2}$
$\therefore\ x = -1, y = 2$
or $x = \frac{1}{2}, y = \frac{1}{2}$

g $y = 4 - x$
sub.
$2x^2 + x(4 - x) + (4 - x)^2 = 22$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$ or 3
$\therefore\ x = -1, y = 5$
or $x = 3, y = 1$

h $x = 2y$
sub.
$(2y)^2 - 4y - y^2 = 0$
$3y^2 - 4y = 0$
$y(3y - 4) = 0$
$y = 0$ or $\frac{4}{3}$
$\therefore\ x = 0, y = 0$
or $x = \frac{8}{3}, y = \frac{4}{3}$

i $y = 3 - \frac{3}{2}x$
sub.
$x^2 + x(3 - \frac{3}{2}x) = 4$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
$x = 2$ or 4
$\therefore\ x = 2, y = 0$
or $x = 4, y = -3$

3.

a sub. $x^2 + (x-4)^2 = 10$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1, 3$

$\therefore$ (1, –3) and (3, –1)

b sub. $y = 17 - 3x$

$x^2 + (17-3x)^2 - 4x - 2(17-3x) - 15 = 0$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x = 4, 6$

$\therefore$ (4, 5) and (6, –1)

c sub.

$4x^2 + 4(2x+2)^2 + 4x - 8(2x+2) - 15 = 0$

$4x^2 + 4x - 3 = 0$

$(2x+3)(2x-1) = 0$

$x = -\frac{3}{2}, \frac{1}{2}$

$\therefore$ $(-\frac{3}{2}, -1)$ and $(\frac{1}{2}, 3)$

4.

sub.

$x^2 + (2x+1)^2 - 8x - 8(2x+1) + 27 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

repeated root $\therefore$ tangent

touch when $x = 2$ $\therefore$ at (2, 5)

Answers – Advanced Skills

1.

$y = \frac{2x+7}{3}$

sub. $2x^2 - (\frac{2x+7}{3})^2 - 7 = 0$

$18x^2 - (2x+7)^2 - 63 = 0$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = -2$ or 4

$\therefore$ $x = -2, y = 1$ or $x = 4, y = 5$

3.

$3^{x-1} = (3^2)^{2y}$ $\therefore$ $x - 1 = 4y$

$(2^3)^{x-2} = (2^2)^{1+y}$ $\therefore$ $3x - 6 = 2 + 2y$

$6x - 16 = 4y$

$\Rightarrow$ $6x - 16 = x - 1$

$x = 3$

$\therefore$ $x = 3, y = \frac{1}{2}$

4.

$(2^2)^{2x} = 2^{y-1}$

$4x = y - 1$ (1)

$(3^2)^{4x} = 3^{y+1}$

$8x = y + 1$ (2)

(1) and (2) $\Rightarrow$ $y = 4x + 1 = 8x - 1$

$4x = 2$

$x = \frac{1}{2}, y = 3$

2.

a subtracting

$-\frac{1}{y} + 2y + 1 = 0$

$-1 + 2y^2 + y = 0$

$2y^2 + y - 1 = 0$

$(2y-1)(y+1) = 0$

$y = -1$ or $\frac{1}{2}$

$\therefore$ $x = -5, y = -1$

or $x = 4, y = \frac{1}{2}$

b $y = x - 5$

sub.

$x(x-5) = 6$

$x^2 - 5x - 6 = 0$

$(x+1)(x-6) = 0$

$x = -1$ or 6

$\therefore$ $x = -1, y = -6$

or $x = 6, y = 1$

c $y = 7 - 4x$

sub.

$\frac{3}{x} - 2(7-4x) + 4 = 0$

$3 - 2x(7-4x) + 4x = 0$

$8x^2 - 10x + 3 = 0$

$(4x-3)(2x-1) = 0$

$x = \frac{1}{2}$ or $\frac{3}{4}$

$\therefore$ $x = \frac{1}{2}, y = 5$

or $x = \frac{3}{4}, y = 4$

Simultaneous Equations Exam Questions

1. Jan 11 Q7

$y = 4 - 10x - x^2 \qquad y = k(4x - 13)$

$k(4x - 13) = 4 - 10x - x^2$

$x^2 + 10x + 4kx - 13k - 4 = 0$

$x^2 + 2(2k + 5)x - (13k + 4) = 0$

2. Jan 13 Q8

$y = 2x^2 - x - 1 \qquad y = k(2x - 3)$

$2x^2 - x - 1 = k(2x - 3)$

$2x^2 - x - 1 = 2kx - 3k$

$2x^2 - 2kx - x + 3k - 1 = 0$

$2x^2 - (2k + 1)x + 3k - 1 = 0$

Topic: B5 Inequalities **Basic Skills**

Mathematics Department

https://youtu.be/wDut-In_7Wg

1. Solve each inequality.

a $2y - 3 > y + 4$ **b** $5p + 1 \leq p + 3$ **c** $x - 2 < 3x - 8$

d $a + 11 \geq 15 - a$ **e** $17 - 2u < 2 + u$ **f** $5 - b \geq 14 - 3b$

g $4x + 23 < x + 5$ **h** $12 + 3y \geq 2y - 1$ **i** $16 - 3p \leq 36 + p$

2. Find the set of values of x for which

a $x^2 - 4x + 3 < 0$ **b** $x^2 - 4 \leq 0$ **c** $15 + 8x + x^2 < 0$ **d** $x^2 + 2x \leq 8$

e $x^2 - 6x + 5 > 0$ **f** $x^2 + 4x > 12$ **g** $x^2 + 10x + 21 \geq 0$ **h** $22 + 9x - x^2 > 0$

i $63 - 2x - x^2 \leq 0$ **j** $x^2 + 11x + 30 > 0$ **k** $30 + 7x - x^2 > 0$ **l** $x^2 + 91 \geq 20x$

3. Solve each inequality.

a $2x^2 - 9x + 4 \leq 0$ **b** $2r^2 - 5r - 3 < 0$ **c** $2 - p - 3p^2 \geq 0$

d $2y^2 + 9y - 5 > 0$ **e** $4m^2 + 13m + 3 < 0$ **f** $9x - 2x^2 \leq 10$

g $a^2 + 6 < 8a - 9$ **h** $x(x + 4) \leq 7 - 2x$ **i** $y(y + 9) > 2(y - 5)$

4. Giving your answers in terms of surds, find the set of values of x for which

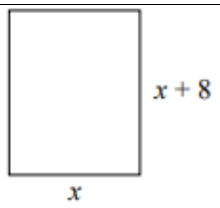
a $x^2 + 2x - 1 < 0$ **b** $x^2 - 6x + 4 > 0$ **c** $11 - 6x - x^2 > 0$ **d** $x^2 + 4x + 1 \geq 0$

5. Find the set of integers, n, for which

$$2n^2 - 5n < 12.$$

Advanced Skills

1.



The diagram shows a rectangular birthday card which is x cm wide and $(x + 8)$ cm tall.

Given that the height of the card is to be at least 50% more than its width,

a show that $x \leq 16$.

Given also that the area of the front of the card is to be at least 180 cm^2,

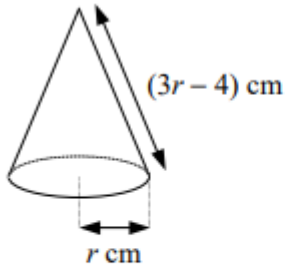
b find the set of possible values of x.

2. Given that $x - y = 8$,

and that $xy \leq 240$,

find the maximum value of $(x + y)$.

3.



A party hat is designed in the shape of a right circular cone of base radius r cm and slant height $(3r-4)$ cm.

Given that the height of the cone must not be more than 24 cm, find the maximum value of r.

Exam Questions *(AQA C1 Questions)*

1. **Jan 11 Q7**

(iii) Solve the inequality $4k^2 + 33k + 29 > 0$. *(4 marks)*

2. **June 11 Q7**

Solve each of the following inequalities:

(a) $2(4-3x) > 5 - 4(x+2)$; *(2 marks)*

(b) $2x^2 + 5x \geqslant 12$. *(4 marks)*

3. **Jan 12 Q6**

A rectangular garden is to have width x metres and length $(x+4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

4. **June 12 Q7a**

(ii) Solve the inequality $3x^2 - 10x + 8 < 0$. *(4 marks)*

Answers – Basic Skills

1.

a $y > 7$

b $4p \le 2$
$p \le \frac{1}{2}$

c $6 < 2x$
$x > 3$

d $2a \ge 4$
$a \ge 2$

e $15 < 3u$
$u > 5$

f $2b \ge 9$
$b \ge \frac{9}{2}$

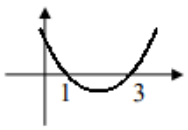
g $3x < -18$
$x < -6$

h $y \ge -13$

i $-20 \le 4p$
$p \ge -5$

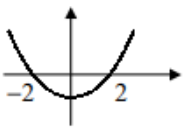
2.

a $(x-1)(x-3) < 0$



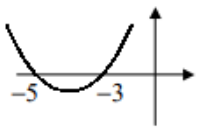
$\therefore 1 < x < 3$

b $(x+2)(x-2) \le 0$



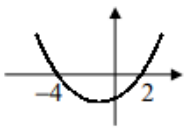
$\therefore -2 \le x \le 2$

c $(x+5)(x+3) < 0$



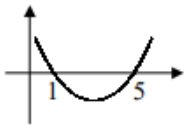
$\therefore -5 < x < -3$

d $x^2 + 2x - 8 \le 0$
$(x+4)(x-2) \le 0$



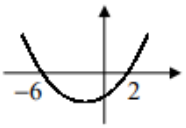
$\therefore -4 \le x \le 2$

e $(x-1)(x-5) > 0$



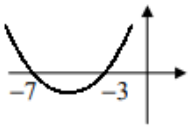
$\therefore x < 1$ or $x > 5$

f $x^2 + 4x - 12 > 0$
$(x+6)(x-2) > 0$



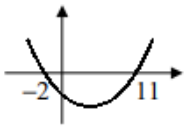
$\therefore x < -6$ or $x > 2$

g $(x+7)(x+3) \ge 0$



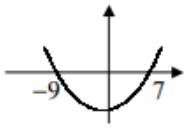
$\therefore x \le -7$ or $x \ge -3$

h $x^2 - 9x - 22 < 0$
$(x+2)(x-11) < 0$



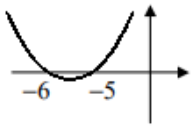
$\therefore -2 < x < 11$

i $x^2 + 2x - 63 \ge 0$
$(x+9)(x-7) \ge 0$



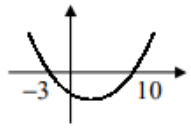
$\therefore x \le -9$ or $x \ge 7$

j $(x+6)(x+5) > 0$



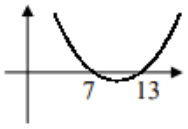
$\therefore x < -6$ or $x > -5$

k $x^2 - 7x - 30 < 0$
$(x+3)(x-10) < 0$



$\therefore -3 < x < 10$

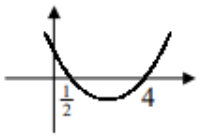
l $x^2 - 20x + 91 \ge 0$
$(x-7)(x-13) \ge 0$



$\therefore x \le 7$ or $x \ge 13$

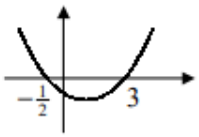
3.

a $(2x-1)(x-4) \le 0$



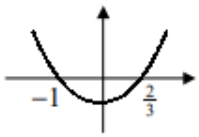
$\therefore \frac{1}{2} \le x \le 4$

b $(2r+1)(r-3) < 0$



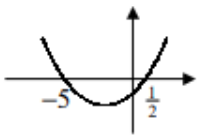
$\therefore -\frac{1}{2} < r < 3$

c $3p^2 + p - 2 \le 0$
$(3p-2)(p+1) \le 0$



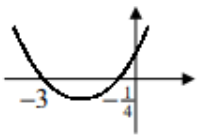
$\therefore -1 \le p \le \frac{2}{3}$

d $(2y-1)(y+5) > 0$



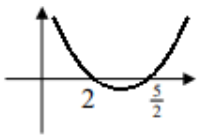
$\therefore y < -5$ or $y > \frac{1}{2}$

e $(4m+1)(m+3) < 0$



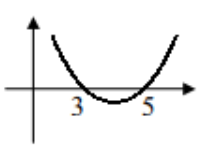
$\therefore -3 < m < -\frac{1}{4}$

f $2x^2 - 9x + 10 \ge 0$
$(2x-5)(x-2) \ge 0$



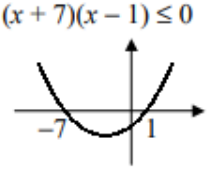
$\therefore x \le 2$ or $x \ge \frac{5}{2}$

g $a^2 - 8a + 15 < 0$
$(a-3)(a-5) < 0$



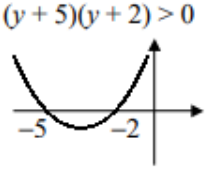
$\therefore 3 < a < 5$

h $x^2 + 4x \le 7 - 2x$
$x^2 + 6x - 7 \le 0$
$(x+7)(x-1) \le 0$



$\therefore -7 \le x \le 1$

i $y^2 + 9y > 2y - 10$
$y^2 + 7y + 10 > 0$
$(y+5)(y+2) > 0$



$\therefore y < -5$ or $y > -2$

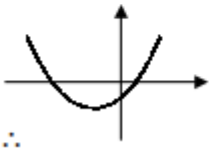
4.

a for critical values

$x = \frac{-2 \pm \sqrt{4+4}}{2}$

$x = \frac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$



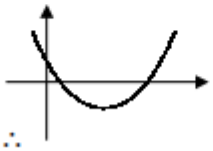
$\therefore$ $-1-\sqrt{2} < x < -1+\sqrt{2}$

b for critical values

$x = \frac{6 \pm \sqrt{36-16}}{2}$

$x = \frac{6 \pm 2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$



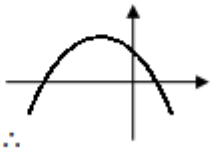
$\therefore$ $x < 3-\sqrt{5}$ or $x > 3+\sqrt{5}$

c for critical values

$x = \frac{6 \pm \sqrt{36+44}}{-2}$

$x = \frac{6 \pm 4\sqrt{5}}{-2}$

$x = -3 \pm 2\sqrt{5}$



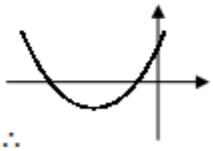
$\therefore$ $-3-2\sqrt{5} < x < -3+2\sqrt{5}$

d for critical values

$x = \frac{-4 \pm \sqrt{16-4}}{2}$

$x = \frac{-4 \pm 2\sqrt{3}}{2}$

$x = -2 \pm \sqrt{3}$

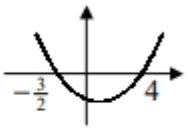


$\therefore$ $x \leq -2-\sqrt{3}$ or $x \geq -2+\sqrt{3}$

5.

$2n^2 - 5n - 12 < 0$

$(2n + 3)(n - 4) < 0$



$-\frac{3}{2} < n < 4$

n integer $\therefore$ $n = -1, 0, 1, 2, 3$

Answers – Advanced Skills

1.

a $(x + 8) \geq 1.5 \times x$

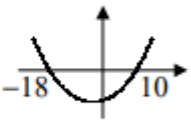
$8 \geq 0.5x$

$x \leq 16$

b $x(x + 8) \geq 180$

$x^2 + 8x - 180 \geq 0$

$(x + 18)(x - 10) \geq 0$



$x \leq -18$ or $x \geq 10$

but $x > 0$ (width > 0)

and $x \leq 16$ $\therefore$ $10 \leq x \leq 16$

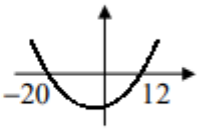
2.

$x = y + 8$

sub. $y(y + 8) \leq 240$

$y^2 + 8y - 240 \leq 0$

$(y + 20)(y - 12) \leq 0$



$-20 \leq y \leq 12$

$x + y = y + 8 + y = 2y + 8$

$\therefore$ max value of $(x + y) = 2(12) + 8 = 32$

3.

let height be h $\therefore$ $h^2 = (3r - 4)^2 - r^2$

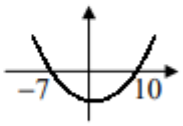
but $h \leq 24$

$\therefore$ $h^2 \leq 24^2$

$(3r - 4)^2 - r^2 \leq 576$

$r^2 - 3r - 70 \leq 0$

$(r + 7)(r - 10) \leq 0$



$-7 \leq r \leq 10$

$\therefore$ maximum value of $r = 10$

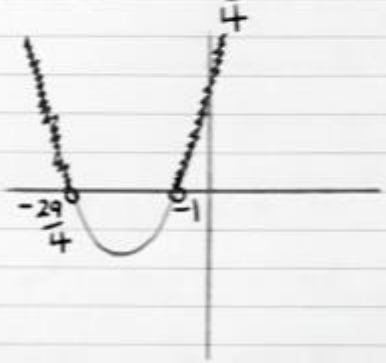
Inequalities Exam Questions Solutions

1. Jan 11 Q7 (ii)

$4R^2 + 33R + 29 > 0$

$(4R + 29)(R+1) > 0$ (m1)

cvs at $R = -\frac{29}{4}$ $R = -1$ (A1)



(m1)

$R < -\frac{29}{4}$ or $R > -1$ (A1)

4

2. June 11 Q7

(a) $2(4 - 3x) > 5 - 4(x+2)$

$8 - 6x > 5 - 4x - 8$ (m1)

$-2x > -11$

$2x < 11$ (x-1 reverse inequality sign)

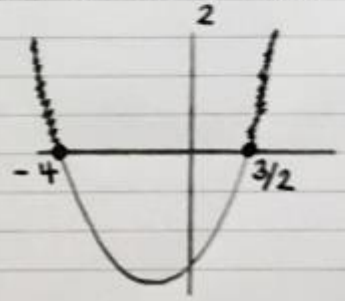
$x < \frac{11}{2}$ (A1)

(b) $2x^2 + 5x \geq 12$

$2x^2 + 5x - 12 \geq 0$

$(2x-3)(x+4) \geq 0$ (m1)

cvs at $x = \frac{3}{2}$ $x = -4$ (A1)

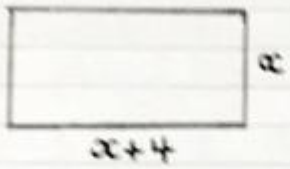


(m1)

6

$x \leq -4$ or $x \geq 3/2$ (A1)

3 Jan 12 Q6



(a) $x + x + x + 4 + x + 4 > 30$

$4x + 8 > 30$

$4x > 22$

$2x > 11$ (B1)

$(x > 11/2)$

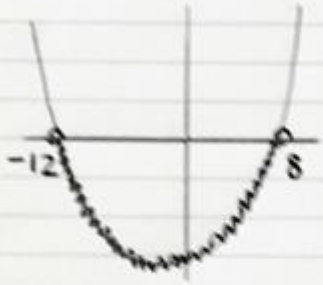
(b) $x(x+4) < 96$

$x^2 + 4x < 96$

$x^2 + 4x - 96 < 0$ (B1)

$(x+12)(x-8) < 0$ (M1)

cvs $x = -12$ $x = 8$ (A1)



(M1)

$-12 < x < 8$ (A1)

(c) $\frac{11}{2} < x < 8$ (B1)

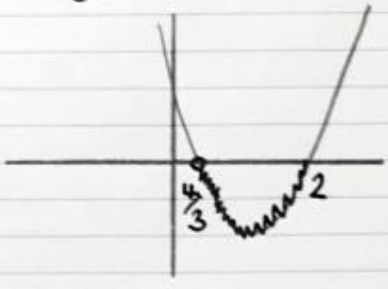
/7

4. June 12 Q7(a)

$3x^2 - 10x + 8 < 0$

$(3x-4)(x-2) < 0$ (M1)

cvs $x = \frac{4}{3}$ $x = 2$ (A1)



(M1)

$\frac{4}{3} < x < 2$ (A1)

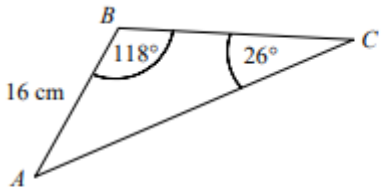
/4

Topic: E1 Triangle Geometry (Trigonometry) **Basic Skills**

https://youtu.be/uVl6TAb0vBg

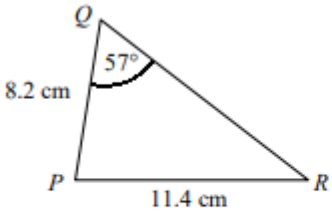
Mathematics Department

1.



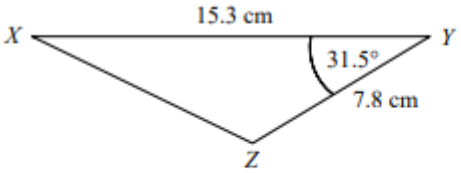
The diagram shows triangle ABC in which $AB = 16$ cm, $\angle ABC = 118°$ and $\angle ACB = 26°$.
Use the sine rule to find the length AC to 3 significant figures.

2.



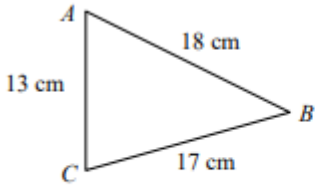
The diagram shows triangle PQR in which $PQ = 8.2$ cm, $PR = 11.4$ cm and $\angle PQR = 57°$.
Use the sine rule to find the size of $\angle PRQ$ in degrees to 1 decimal place.

3.



The diagram shows triangle XYZ in which $XY = 15.3$ cm, $YZ = 7.8$ cm and $\angle XYZ = 31.5°$.
Use the cosine rule to find the length XZ.

4.

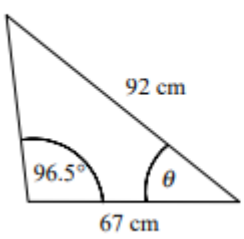


The diagram shows triangle ABC in which $AB = 18$ cm, $AC = 13$ cm and $BC = 17$ cm.
Use the cosine rule to find the size of $\angle ACB$.

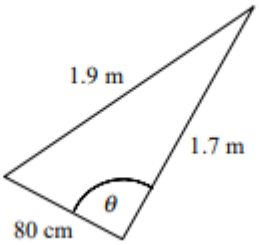
5.

Find the angle θ in each triangle.

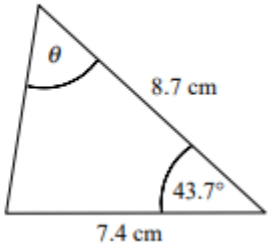
a



b



c



Advanced Skills

1.

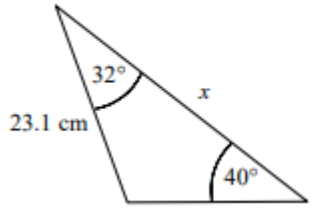
In triangle ABC, $AB = 16.2$ cm, $BC = 12.3$ cm and $\angle BAC = 37°$.

Find the two possible sizes of $\angle ACB$ and the corresponding lengths of AC.

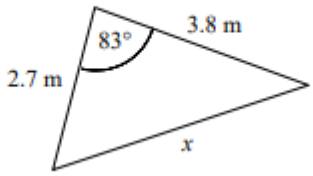
2.

Find the length x in each triangle.

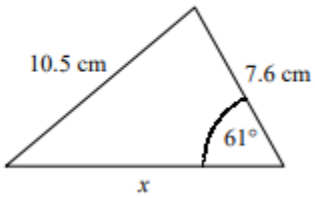
a



b



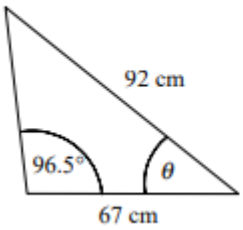
c



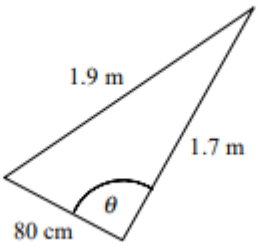
3.

Find the angle θ in each triangle.

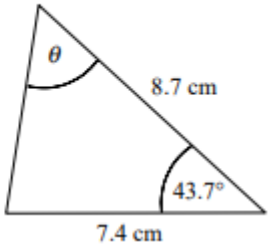
a



b



c



4.

Joanne walks 4.2 miles on a bearing of 138°. She then walks 7.8 miles on a bearing of 251°.

a Calculate how far Joanne is from the point where she started.

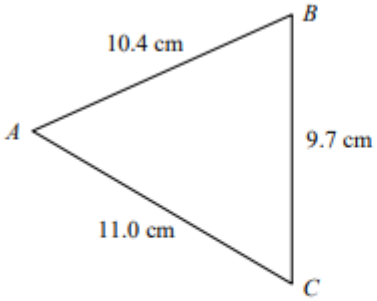
b Find, as a bearing, the direction in which Joanne would have to walk in order to return to the point where she started.

5.

A ferry and a cargo ship are both approaching the same port. The ferry is 3.2 km from the port on a bearing of 076° and the cargo ship is 6.9 km from the port on a bearing of 323°.

Find the distance between the two vessels and the bearing of the cargo ship from the ferry.

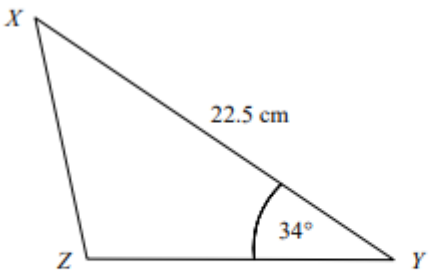
6.



The diagram shows triangle ABC in which $AB = 10.4$ cm, $AC = 11.0$ cm and $BC = 9.7$ cm.

Find the area of the triangle to 3 significant figures.

7.



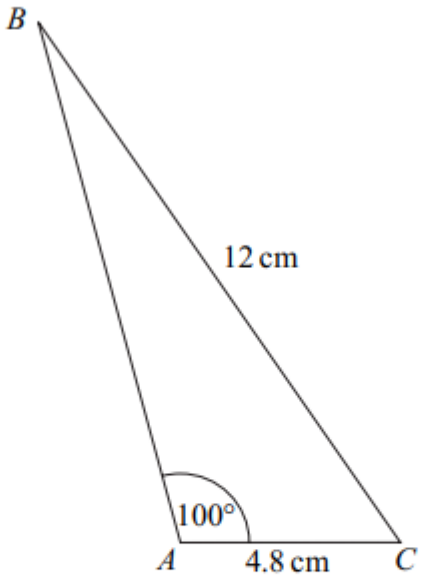
The diagram shows triangle XYZ in which $XY = 22.5$ cm and $\angle XYZ = 34°$.

Given that the area of the triangle is 100 cm^2, find the length XZ.

Exam Questions *(AQA C2 Questions)*

1. June 2006 Q2

The diagram shows a triangle ABC.



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

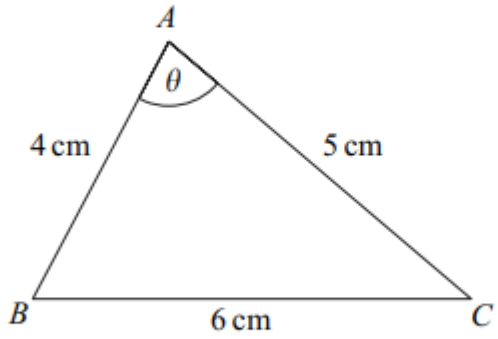
The size of the angle BAC is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

2. Jan 2007 Q4 (adapted)

The triangle ABC, shown in the diagram, is such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm. The angle BAC is θ.

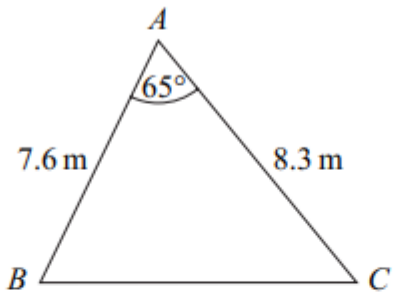


(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

3. **June 2008 Q4**

The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

(a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

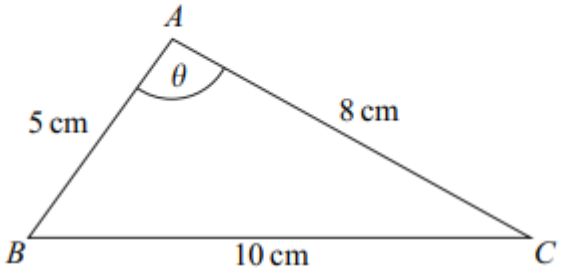
(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

4. **Jan 2011 Q3**

The triangle ABC, shown in the diagram, is such that $AB = 5$ cm, $AC = 8$ cm, $BC = 10$ cm and angle $BAC = \theta$.



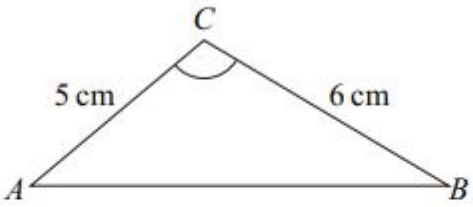
(a) Show that $\theta = 97.9°$, correct to the nearest 0.1°. *(3 marks)*

(b) (i) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

(ii) The line through A, perpendicular to BC, meets BC at the point D. Calculate the length of AD, giving your answer, in cm, to three significant figures. *(3 marks)*

5. **Jan 2013 Q3**

The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 6 cm respectively.

The area of triangle ABC is 12.5 cm^2, and angle ACB is **obtuse**.

(a) Find the size of angle ACB, giving your answer to the nearest 0.1°. *(3 marks)*

(b) Find the length of AB, giving your answer to two significant figures. *(3 marks)*

Answers – Basic Skills

1.

$$\frac{AC}{\sin 118} = \frac{16}{\sin 26}$$
$$AC = \frac{16 \times \sin 118}{\sin 26}$$
$$= 32.2 \text{ cm}$$

2.

$$\frac{\sin \angle PRQ}{8.2} = \frac{\sin 57}{11.4}$$
$$\sin \angle PRQ = \frac{8.2 \times \sin 57}{11.4} = 0.6033$$
$$\angle PRQ = 37.1°$$

3.

$$XZ^2 = 7.8^2 + 15.3^2 - (2 \times 7.8 \times 15.3 \times \cos 31.5°)$$
$$= 91.422$$
$$XZ = 9.56 \text{ cm (3sf)}$$

4.

$$18^2 = 13^2 + 17^2 - (2 \times 13 \times 17 \times \cos \angle ACB)$$
$$\cos \angle ACB = \frac{13^2 + 17^2 - 18^2}{2 \times 13 \times 17}$$
$$= 0.3032$$
$$\angle ACB = 72.4° \text{ (1dp)}$$

5.

a area

$$= \tfrac{1}{2} \times 2.1 \times 3.4 \times \sin 66$$
$$= 3.26 \text{ m}^2 \text{ (3sf)}$$

b area

$$= \tfrac{1}{2} \times 35 \times 68 \times \sin 116$$
$$= 1070 \text{ cm}^2 \text{ (3sf)}$$

c

$$\frac{\sin \alpha}{5.8} = \frac{\sin 72.4}{6.5}$$
$$\sin \alpha = \frac{5.8 \times \sin 72.4}{6.5} = 0.8505$$
$$\alpha = 58.270$$
$$\beta = 180 - (72.4 + \alpha) = 49.330$$
area
$$= \tfrac{1}{2} \times 5.8 \times 6.5 \times \sin 49.330$$
$$= 14.3 \text{ cm}^2 \text{ (3sf)}$$

Answers – Advanced Skills

1.

$$\frac{\sin\angle ACB}{16.2} = \frac{\sin 37}{12.3}$$

$$\sin\angle ACB = \frac{16.2\times\sin 37}{12.3} = 0.7926$$

$\angle ACB = 52.4$ or $180 - 52.4 = 52.4$ or 127.6

$\angle ABC = 180 - (37 + \angle ACB) = 90.568$ or 15.432

$$\frac{AC}{\sin\angle ABC} = \frac{12.3}{\sin 37}$$

$$AC = \frac{12.3\times\sin\angle ABC}{\sin 37} = 20.4 \text{ or } 5.4$$

$\therefore \angle ACB = 52.4°, AC = 20.4$ cm or $\angle ACB = 127.6°, AC = 5.4$ cm (all 1dp)

2.

a $\alpha = 180 - (40 + 32) = 108$

$$\frac{x}{\sin 108} = \frac{23.1}{\sin 40}$$

$$x = \frac{23.1\times\sin 108}{\sin 40}$$

$x = 34.2$ cm (3sf)

b $x^2 = 2.7^2 + 3.8^2 - (2\times 2.7\times 3.8\times\cos 83)$

$x^2 = 19.229$

$x = 4.39$ m (3sf)

c

$$\frac{\sin\alpha}{7.6} = \frac{\sin 61}{10.5}$$

$$\sin\alpha = \frac{7.6\times\sin 61}{10.5} = 0.6331$$

$\alpha = 39.276$

$\beta = 180 - (61 + 39.276) = 79.724$

$$\frac{x}{\sin 79.724} = \frac{10.5}{\sin 61}$$

$$x = \frac{10.5\times\sin 79.724}{\sin 61}$$

$x = 11.8$ cm (3sf)

3.

a

$$\frac{\sin\alpha}{67} = \frac{\sin 96.5}{92}$$

$$\sin\alpha = \frac{67\times\sin 96.5}{92}$$

$\sin\alpha = 0.7236$

$\alpha = 46.351$

$\theta = 180 - 96.5 - \alpha$

$\theta = 37.1°$ (1dp)

b $1.9^2 = 0.8^2 + 1.7^2 - (2\times 0.8\times 1.7\times\cos\theta)$

$$\cos\theta = \frac{0.8^2 + 1.7^2 - 1.9^2}{2\times 0.8\times 1.7}$$

$\cos\theta = -0.02941$

$\theta = 91.7°$ (1dp)

c $l^2 = 7.4^2 + 8.7^2 - (2\times 7.4\times 8.7\times\cos 43.7)$

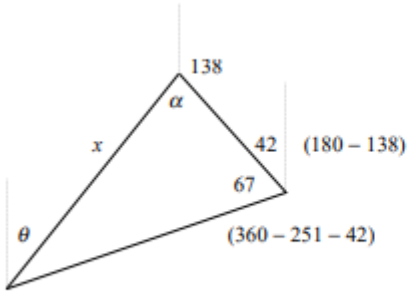
$l^2 = 37.3608, l = 6.1123$

$$\frac{\sin\theta}{7.4} = \frac{\sin 43.7}{6.1123}$$

$$\sin\theta = \frac{7.4\times\sin 43.7}{6.1123} = 0.8364$$

$\theta = 56.8°$ (1dp)

4.



a $x^2 = 4.2^2 + 7.8^2 - (2\times 4.2\times 7.8\times\cos 67)$

$x^2 = 52.879$

$x = 7.27$ miles (3sf)

b

$$\frac{\sin\alpha}{7.8} = \frac{\sin 67}{7.2718}$$

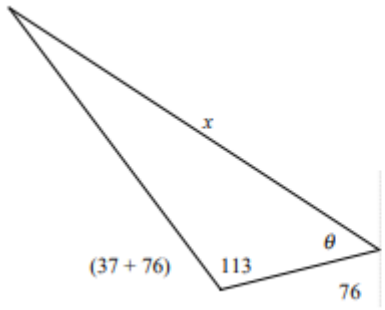
$$\sin\alpha = \frac{7.8\times\sin 67}{7.2718} = 0.9874$$

$\alpha = 80.882$

$\theta = 138 + \alpha - 180 = 38.882$

bearing = 039° (nearest degree)

5.



$x^2 = 3.2^2 + 6.9^2 - (2 \times 3.2 \times 6.9 \times \cos 113)$
$x^2 = 75.105$
$x = 8.67$ km (3sf)

$\frac{\sin\theta}{6.9} = \frac{\sin 113}{8.666}$
$\sin\theta = \frac{6.9 \times \sin 113}{8.666} = 0.7329$
$\theta = 47.130$
bearing $= 180 + 76 + \theta = 303°$ (nearest degree)

6.

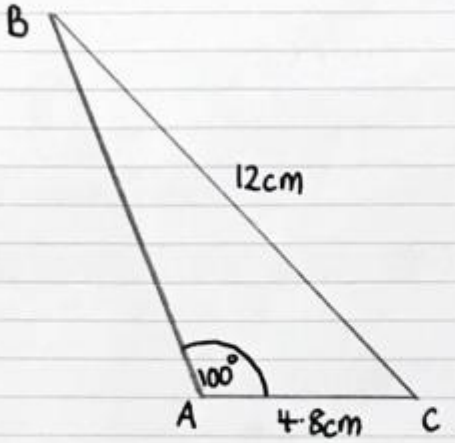
$9.7^2 = 10.4^2 + 11.0^2 - (2 \times 10.4 \times 11.0 \times \cos \angle BAC)$
$\cos \angle BAC = \frac{10.4^2 + 11.0^2 - 9.7^2}{2 \times 10.4 \times 11.0} = 0.5903$
$\angle BAC = 53.819$
area $= \frac{1}{2} \times 10.4 \times 11.0 \times \sin 53.819 = 46.2$ cm^2

7.

$\frac{1}{2} \times 22.5 \times YZ \times \sin 34 = 100$
$YZ = \frac{200}{22.5 \times \sin 34} = 15.896$
$XZ^2 = 22.5^2 + 15.896^2 - (2 \times 22.5 \times 15.896 \times \cos 34)$
$= 165.906$
$XZ = 12.9$ cm (3sf)

Triangle Geometry Exam Questions

1. June 2006 Q2



(a) $\frac{\sin A\hat{B}C}{4.8} = \frac{\sin 100}{12}$ (M1)

$\sin A\hat{B}C = 0.4 \sin 100$ (M1)

$A\hat{B}C = \sin^{-1}(0.4 \sin 100)$

$A\hat{B}C = 23.19882755$

$A\hat{B}C = 23.2$ correct to nearest $0.1°$ (A1)

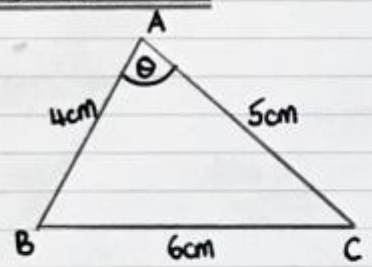
(b) angle $A\hat{C}B = 180 - 100 - 23.2$
$= 56.8°$ (M1)

area of $\Delta = \frac{1}{2} \times 4.8 \times 12 \times \sin 56.8$ (M1)

$= 24.1$ cm^2 to 3.s.f. (A1)

6

2. Jan 2007 Q4



(a) Using cosine rule (M1)

$6^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$

$36 = 41 - 40\cos\theta$

$40\cos\theta = 5$ (M1)

$\cos\theta = \frac{5}{40}$

$\cos\theta = \frac{1}{8}$ (A1)

(c) $\theta = \cos^{-1}\left(\frac{1}{8}\right)$

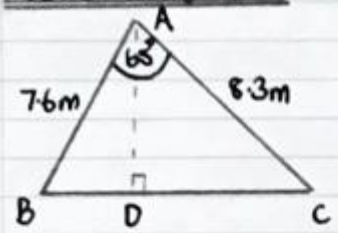
$\theta = 82.8°$ (M1)

$A = \frac{1}{2} \times 4 \times 5 \times \sin 82.8°$

Area $= 9.92\text{cm}^2$ to 3.s.f. (A1)

5

3. June 2008 Q4



(a) Using cosine rule

$BC^2 = 7.6^2 + 8.3^2 - 2(7.6)(8.3)\cos 65°$ (M1)

$BC^2 = 73.33248...$ (M1)

$BC = 8.5634...$

$BC = 8.56\text{m}$ to 3.s.f. (A1)

(b) $A = \frac{1}{2} \times 7.6 \times 8.3 \sin 65$ (M1)

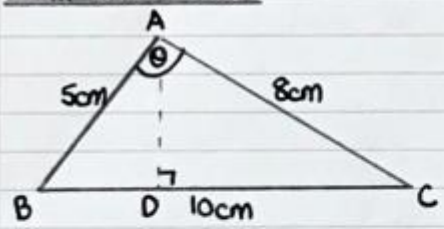
$= 28.6\text{m}^2$ to 3.s.f. (A1)

(c) $28.6 = \frac{1}{2} \times 8.56 \times AD$ (M1) (M1)

$AD = 6.7\text{m}$ to nearest 0.1m (A1)

8

4. Jan 2011 Q3



(a) Using cosine rule

$10^2 = 5^2 + 8^2 - 2(5)(8)\cos\theta$ (M1)

$100 = 89 - 80\cos\theta$ (M1)

$80\cos\theta = -11$

$\cos\theta = \frac{-11}{80}$

$\theta = 97.9032...$

$\theta = 97.9°$ to nearest $0.1°$ (A1)

(b) (i) Area $= \frac{1}{2} \times 5 \times 8 \times \sin 97.9°$ (M1)

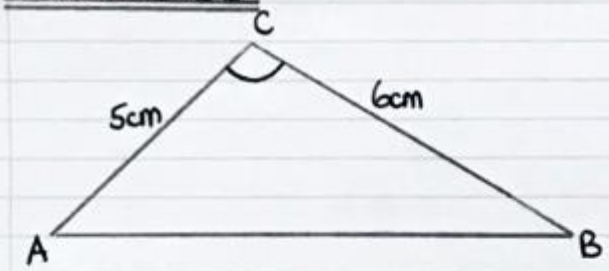
$= 19.8\text{cm}^2$ to 3.s.f. (A1)

(ii) $19.8 = \frac{1}{2} \times 10 \times AD$ (M1)(M1)

$AD = 3.96$ cm to 3.s.f. (A1)

8

5. Jan 2013 Q3



(a) $12.5 = \frac{1}{2} \times 5 \times 6 \times \sin C$ (M1)

$\frac{12.5}{15} = \sin C$ (A1)

$C = 56.4°$

but $A\hat{C}B$ is obtuse

$\therefore A\hat{C}B = 180 - 56.4$

$= 123.6°$ (A1)

(b) Using cosine rule

$AB^2 = 5^2 + 6^2 - 2(5)(6)\cos 123.6°$ (M1)(M1)

$AB^2 = 94.203...$

$AB = 9.7$ cm to 2.s.f. (A1)

6

Year 12 Initial Test for Mathematics

Write out the solutions to each of the following questions.
*Show full working, **without** the use of a calculator.*

Your initial test for mathematics will look exactly like this so use the videos and worksheets to ensure you are able to do the following skills with the layout expected.

Practice 1 (No Calculator)

B1 Indices

1.	Evaluate $\left(\frac{8}{125}\right)^{-2/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$	**3.**	Solve $9^{x-2} = 27$	**4.**	Solve $16^x = 4^{1-x}$

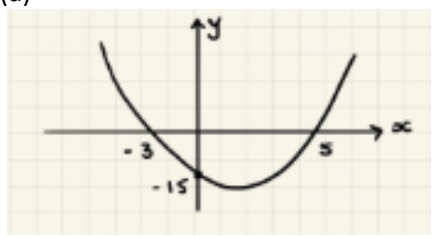
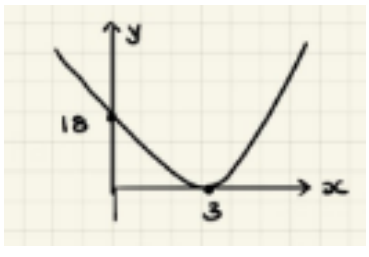
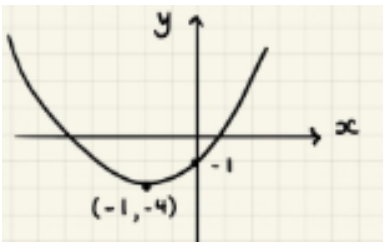
B2 Surds

1.	Simplify $\sqrt{72}$	**2.**	Expand and simplify $(2\sqrt{7} - 5\sqrt{3})\,(3\sqrt{7} + 4\sqrt{3})$	**3.**	Rationalise the denominator $\frac{11}{2\sqrt{5}}$	**4.**	Rationalise the denominator $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}}$

B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.		
(a) (i) $x^2 + 3x - 28 = 0$	(b) (i) $x^2 - 6x + 9 = 0$	(c) (i) $2x^2 - 21x + 27 = 0$
(a) (ii) Sketch $y = x^2 + 3x - 28$	(b) (ii) Sketch $y = x^2 - 6x + 9$	(c) (ii) Sketch $y = 2x^2 - 21x + 27$

2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.		
(a) (i) $x^2 + 4x - 7 = 0$	(b) (i) $11 + 8x - x^2 = 0$	(c) (i) $3x^2 - 12x + 2 = 0$
(ii) Write $y = x^2 + 4x - 7$ in the form $y = a(x + b)^2 + c$	(ii) Write $y = 11 + 8x - x^2$in the form $y = a(x + b)^2 + c$	(ii) Write $y = 3x^2 - 12x + 2$ in the form $y = a(x + b)^2 + c$
(iii) Sketch $y = x^2 + 4x - 7$	(iii) Sketch $y = 11 + 8x - x^2$	(iii) Sketch $y = 3x^2 - 12x + 2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$		
(a) 	(b) 	(c) 

B4 Simultaneous Equations

1.	Solve $3x + 3y = -4$ $5x - 2y = 5$	**2.**	Solve $y = x - 6$ $\frac{1}{2}x - y = 4$	**3.**	Solve $3x^2 - x - y^2 = 0$ $x + y = 1$

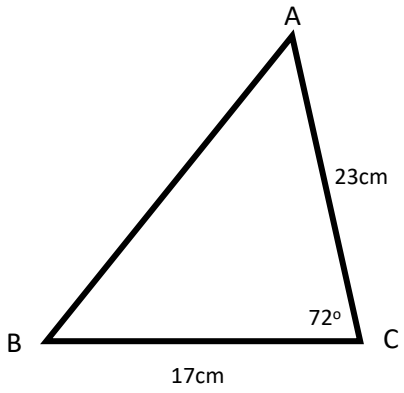
B5 Inequalities

Find the set of values for which…

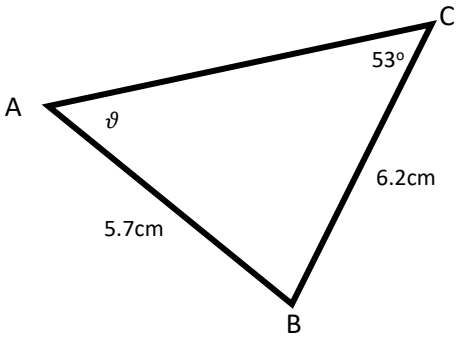
1.	$3(1 - 2t) \leq t - 4$	**2.**	$2x^2 - 9x + 4 \leq 0$	**3.**	$2y + 3 < 3y(y - 2)$

E1 Triangle Geometry

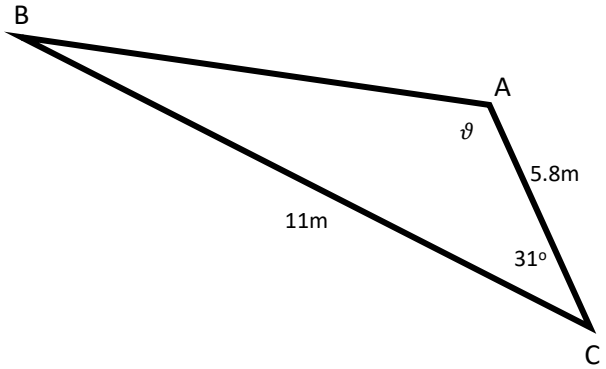
1. Calculate the length AB



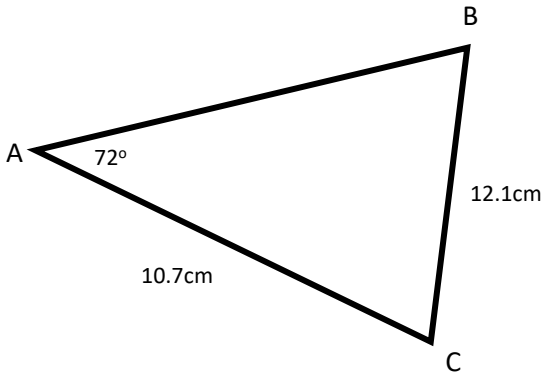
2. Calculate the angle ϑ



3. Calculate the length AB and the obtuse angle ϑ



4. Calculate the area of the triangle ABC



Practice 1

B1 Indices

1. $\left(\frac{8}{125}\right)^{-2/3}$

$= \left(\frac{125}{8}\right)^{2/3}$

$= \left(\frac{5}{2}\right)^{2}$ M1

$= \frac{25}{4}$ A1

2. $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/3}}{x^2}$ M1

$= \frac{x^{5/6}}{x^2}$ A1

$= x^{-7/6}$ A1

3. $9^{x-2} = 27$

$(3^2)^{x-2} = 3^3$ M1

$3^{2x-4} = 3^3$

$2x - 4 = 3$ M1

$2x = 7$

$x = 7/2$ A1

4. $16^{x} = 4^{1-x}$

$(4^2)^{x} = 4^{1-x}$ M1

$4^{2x} = 4^{1-x}$

$2x = 1 - x$ M1

$3x = 1 \Rightarrow x = 1/3$ A1

11

B2 Surds

1. $\sqrt{72}$

$= \sqrt{36 \times 2}$

$= 6\sqrt{2}$ A1

2. $(2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$

$42 + 8\sqrt{21} - 15\sqrt{21} - 60$ M1 A1

$-7\sqrt{21} - 18$ A1

3. $\frac{11}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ M1

$= \frac{11\sqrt{5}}{10}$ A1

4. $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}} \quad \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$ M1

$= \frac{16 - 8\sqrt{5} - 6\sqrt{5} + 15}{4 - 5}$ A1 A1

$= \frac{31 - 14\sqrt{5}}{-1} = 14\sqrt{5} - 31$ A1

10

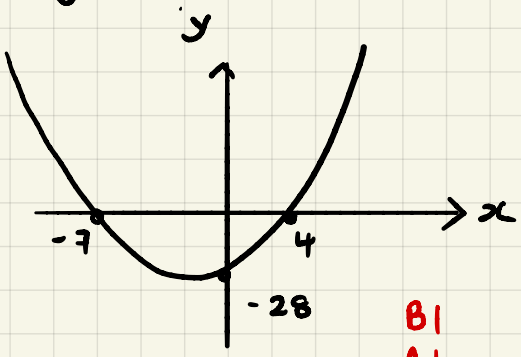
B3 Quadratics

1. (a) (i) $x^2 + 3x - 28 = 0$

$(x + 7)(x - 4) = 0$ M1

$x = -7$ or $x = 4$ A1

(ii) $y = x^2 + 3x - 28$



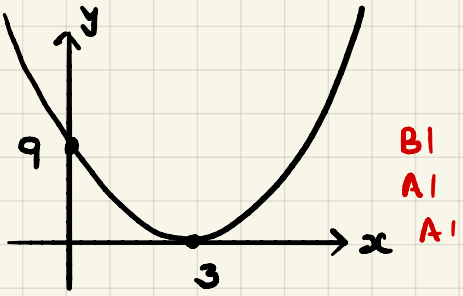
B1
A1
A1

(b) (i) $x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$ M1

A1 $x = 3$ (repeated)

(ii) $y = x^2 - 6x + 9$



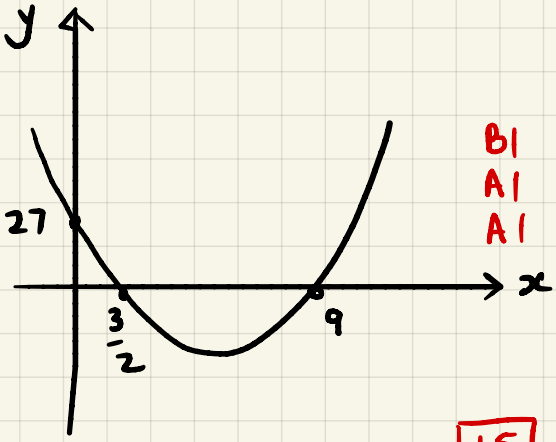
B1
A1
A1

(c) (i) $2x^2 - 21x + 27 = 0$

$(2x - 3)(x - 9) = 0$ M1

$x = 3/2$ $x = 9$ A1

(ii) $y = 2x^2 - 21x + 27$



B1
A1
A1

15

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

2. (a) (i) $x^2 + 4x - 7 = 0$

$(x+2)^2 - 4 - 7 = 0$ M1

$(x+2)^2 = 11$

$x + 2 = \pm\sqrt{11}$

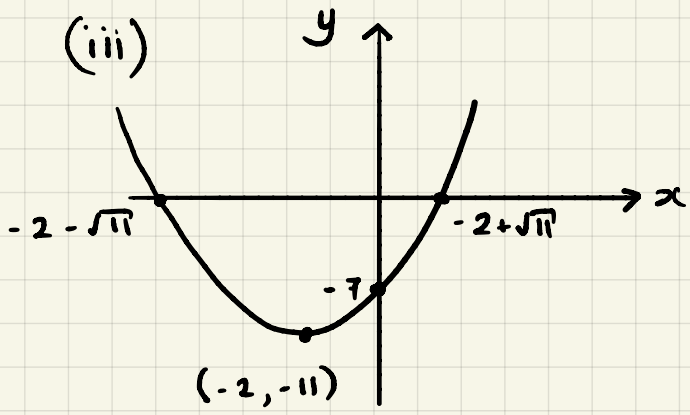
$x = -2 \pm \sqrt{11}$ A1

Graphs
- B1 Shape
- A1 Vertex
- A1 intersections x-axis
- A1 intersections y-axis

(ii) $y = x^2 + 4x - 7$

$y = (x+2)^2 - 11$ B1

(iii)



(b) (i) $11 + 8x - x^2 = 0$

$-(x^2 - 8x - 11) = 0$ M1

$-[(x-4)^2 - 16 - 11] = 0$ M1

$-(x-4)^2 + 27 = 0$

$(x-4)^2 = 27$

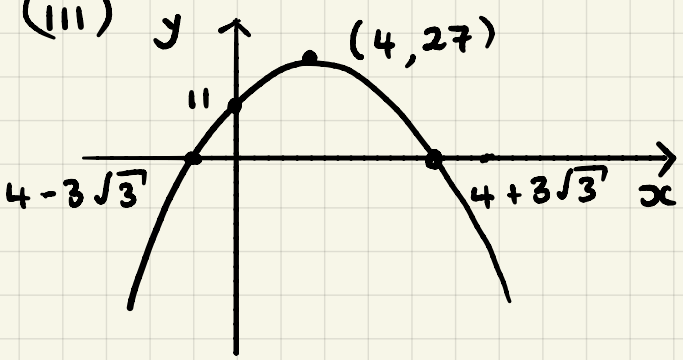
$x - 4 = \pm 3\sqrt{3}$

$x = 4 \pm 3\sqrt{3}$ A1

(ii) $y = 11 + 8x - x^2$

$y = 27 - (x-4)^2$ B1

(iii)



(c) (i) $3x^2 - 12x + 2 = 0$

$3\left[x^2 - 4x + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - 4 + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - \frac{10}{3}\right] = 0$

$3(x-2)^2 - 10 = 0$

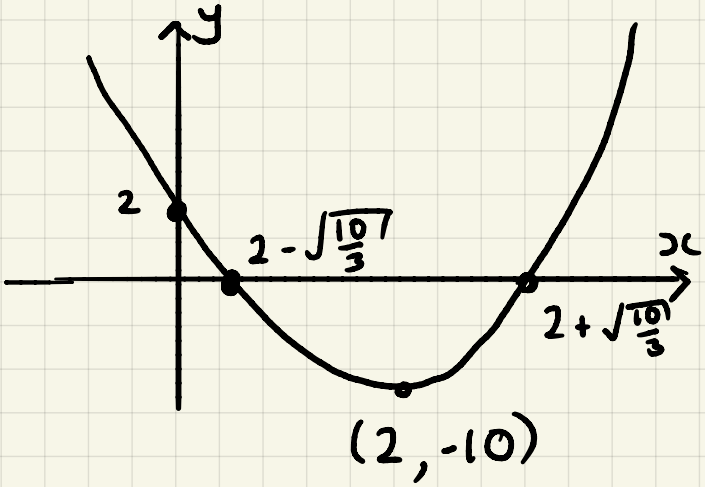
$(x-2)^2 = \frac{10}{3}$

$x - 2 = \pm\sqrt{\frac{10}{3}}$

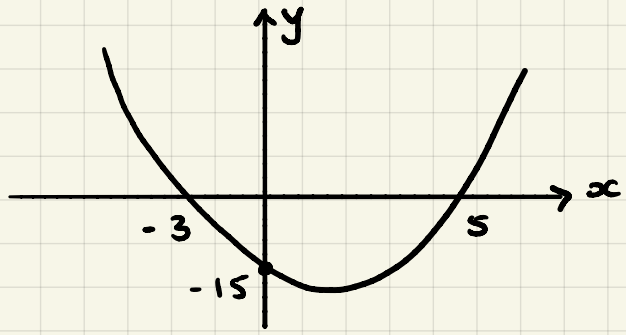
$x = 2 \pm \sqrt{\frac{10}{3}}$ A1

(ii) $y = 3x^2 - 12x + 2$

$y = 3(x-2)^2 - 10$ B1



3. (a)



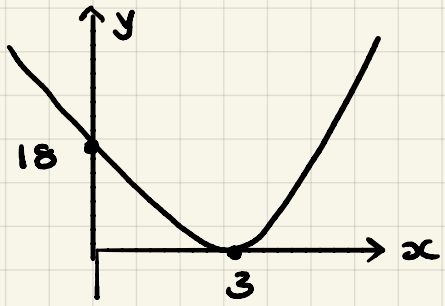
$y = k(x+3)(x-5)$ M1

$-15 = k(3)(-5) \Rightarrow k = 1$ A1

$y = (x+3)(x-5)$

$y = x^2 - 2x - 15$ A1

(b)



$y = k(x-3)^2$ M1

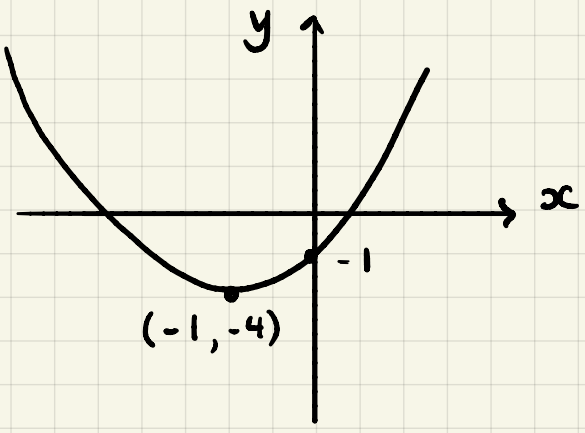
$18 = k(-3)^2 \Rightarrow k = 2$ A1

$y = 2(x-3)^2$

$y = 2(x^2 - 6x + 9)$

$y = 2x^2 - 12x + 18$ A1

(c)



$y = k(x+1)^2 - 4$ M1

(0, -1)

$-1 = k(1)^2 - 4$

$\Rightarrow k = 3$ A1

$y = 3(x+1)^2 - 4$

$y = 3(x^2 + 2x + 1) - 4$

$y = 3x^2 + 6x - 1$ A1

[9]

B4 Simultaneous Equations

1. $3x + 3y = -4$ $\quad 6x + 6y = -8$

$5x - 2y = 5$ $\quad 15x - 6y = 15$ add $\quad$ M1

$21x = 7$

$x = 1/3$ A1 $\quad 3(1/3) + 3y = -4$

$3y = -5$

$x = 1/3, \ y = -5/3$ A1

2. $y = x - 6$

$\frac{1}{2}x - y = 4$

$\frac{1}{2}x - (x - 6) = 4$ M1

$\frac{1}{2}x - x + 6 = 4$

$-\frac{1}{2}x = -2$

$x = 4$ A1 $\quad y = 4 - 6$

$y = -2$

$x = 4, \ y = -2$ A1

3. $3x^2 - x - y^2 = 0$ $\quad x + y = 1$

$y = 1 - x$

$3x^2 - x - (1-x)^2 = 0$ M1

$3x^2 - x - (1 - 2x + x^2) = 0$

$3x^2 - x - 1 + 2x - x^2 = 0$

$2x^2 + x - 1 = 0$ A1

$(2x - 1)(x + 1) = 0$

$x = 1/2$ $\quad x = -1$ A1

$y = 1 - 1/2$ $\quad y = 1 - -1$

$x = 1/2$ $y = 1/2$ A1 $\quad x = -1$, $y = 2$ A1

11

B5 Inequalities

1. $3(1 - 2t) \leq t - 4$

$3 - 6t \leq t - 4$

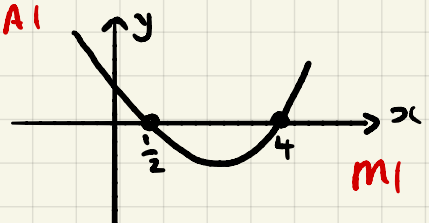
$7 \leq 7t$ M1

$t \geq 1$ A1

2. $2x^2 - 9x + 4 \leq 0$

$(2x - 1)(x - 4) \leq 0$ M1

CVs $x = 1/2$ $\quad x = 4$ A1



M1

$\frac{1}{2} \leq x \leq 4$ A1

3. $2y + 3 < 3y(y - 2)$

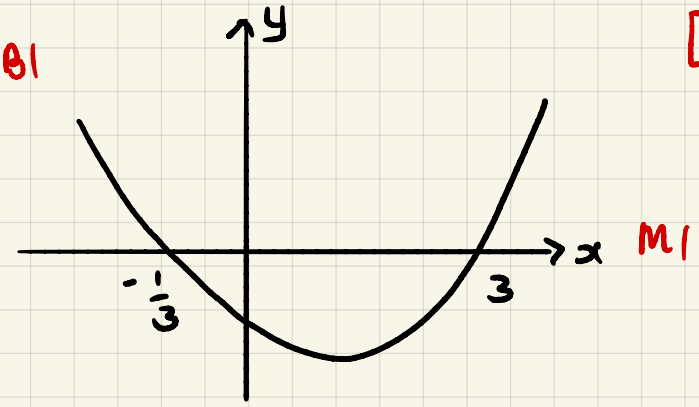
$2y + 3 < 3y^2 - 6y$

$0 < 3y^2 - 8y - 3$ B1

$3y^2 - 8y - 3 > 0$

$(3y + 1)(y - 3) > 0$ M1

CVs $y = -1/3$ $\quad y = 3$ A1



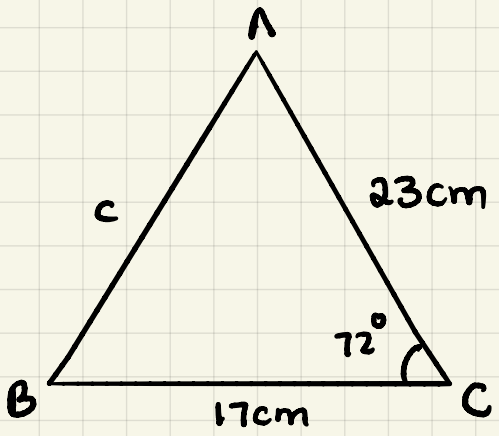
11

M1

$y < -1/3$ $\quad$ or $\quad y > 3$ A1

E1 Triangle Geometry

1.



$c^2 = a^2 + b^2 - 2ab\cos C$

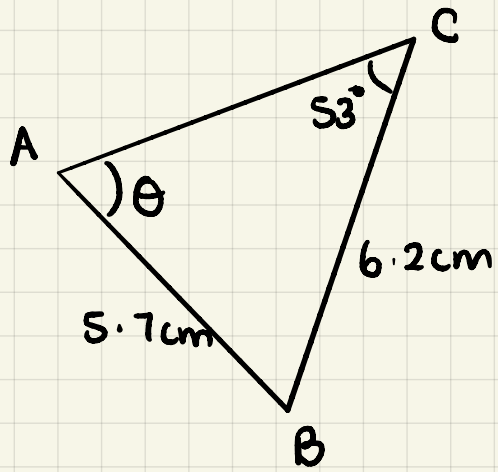
$c^2 = 17^2 + 23^2 - 2(17)(23)\cos 72^\circ$

M1

$c^2 = 576.35$

$AB = 24.0\text{cm}$ A1

2.

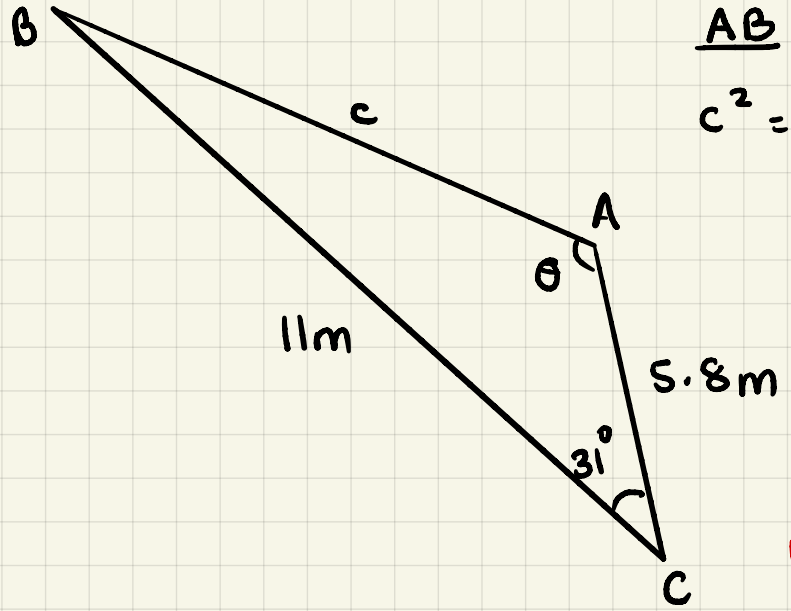


$\frac{\sin\theta}{6.2} = \frac{\sin 53}{5.7}$ M1

$\theta = \sin^{-1}\left(\frac{6.2\sin 53}{5.7}\right)$

$\theta = 60.3^\circ$ A1

3.



AB

$c^2 = 5.8^2 + 11^2 - 2(5.8)(11)\cos 31$

$c^2 = 45.27$ M1

$AB = 6.7\text{m}$ A1

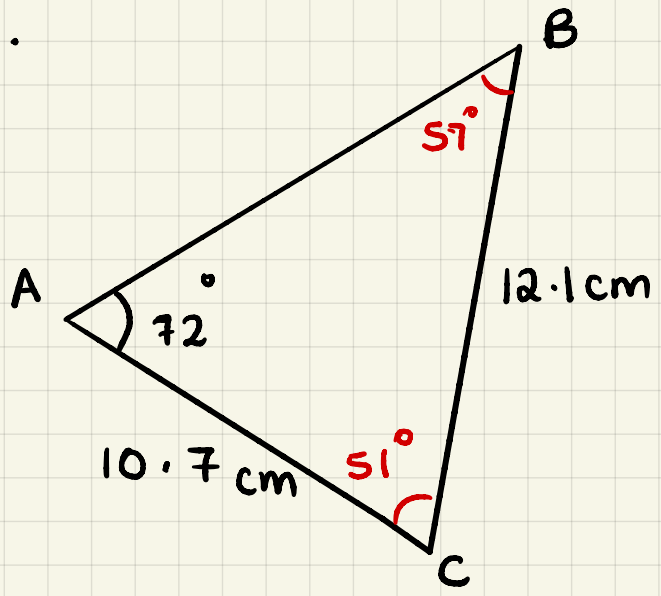
θ

$\cos\theta = \frac{5.8^2 + 6.7^2 - 11^2}{2(5.8)(6.7)}$ M1

$\theta = \cos^{-1}(-0.546)$ A1

$\theta = 123^\circ$

4.



$$\frac{\sin B}{10.7} = \frac{\sin 72^\circ}{12.1}$$ M1

$$B = \sin^{-1}\left(\frac{10.7 \sin 72^\circ}{12.1}\right)$$

$$\theta = 57^\circ$$ A1

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(10.7)(12.1)\sin 51^\circ$$ M1

$$= 50.3 cm^2$$ A1

12

Year 12 Initial Test for Mathematics

Write out the solutions to each of the following questions.
*Show full working, **without** the use of a calculator.*

Your initial test for mathematics will look exactly like this so use the videos and worksheets to ensure you are able to do the following skills with the layout expected.

Practice 2 (No Calculator)

B1 Indices

1.	Evaluate $\left(3\frac{3}{8}\right)^{-1/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x}\times\sqrt[5]{x}}{x^2}$	**3.**	Solve $3^{3x-2}=\sqrt[3]{9}$	**4.**	Solve $\left(\frac{1}{2}\right)^{1-x}=\left(\frac{1}{8}\right)^{2x}$

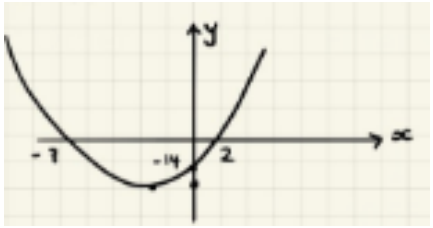
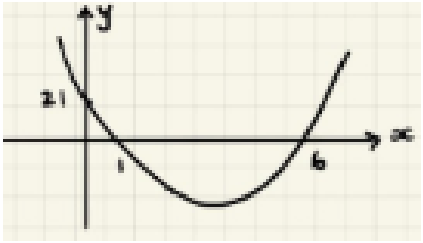
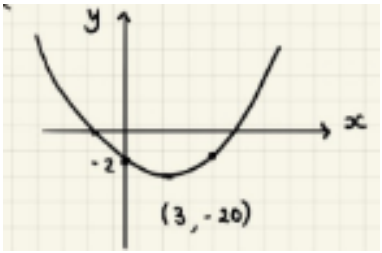
B2 Surds

1.	Simplify $\sqrt{80}$	**2.**	Expand and simplify $(7-3\sqrt{5})(3\sqrt{5}-2)$	**3.**	Rationalise the denominator $\frac{7}{5\sqrt{3}}$	**4.**	Rationalise the denominator $\frac{3+5\sqrt{11}}{7-\sqrt{11}}$

B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.		
(a) (i) $x^2-13x+40=0$	(b) (i) $x^2+5x=0$	(c) (i) $6x^2+5x-4=0$
(a) (ii) Sketch $y=x^2-13x+40$	(b) (ii) Sketch $y=x^2+5x$	(c) (ii) Sketch $y=6x^2+5x-4$

2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.		
(a) (i) $x^2+2x-20=0$	(b) (i) $-11+8x-x^2=0$	(c) (i) $3x^2-18x+2=0$
(ii) Write $y=x^2+2x-20$ in the form $y=a(x+b)^2+c$	(ii) Write $y=-11+8x-x^2$in the form $y=a(x+b)^2+c$	(ii) Write $y=3x^2-18x+2$ in the form $y=a(x+b)^2+c$
(iii) Sketch $y=x^2+2x-20$	(iii) Sketch $y=-11+8x-x^2$	(iii) Sketch $y=3x^2-18x+2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y=ax^2+bx+c$		
(a) 	(b) 	(c) 

B4 Simultaneous Equations

1.	Solve $3x - 4y = 16$ $2x + 12y = 7$	**2.**	Solve $3y = 2x - 8$ $4x + y = -5$	**3.**	Solve $3x^2 - xy + y^2 = 36$ $x - 2y = 10$

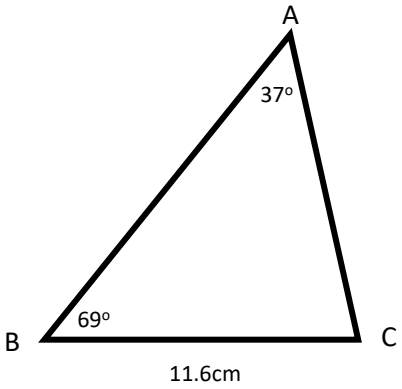
B5 Inequalities

Find the set of values for which…

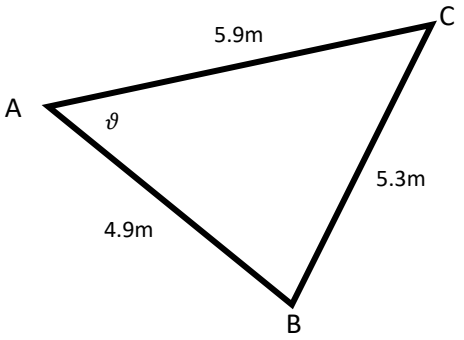
1.	$4(5 - 2y) \geq 3(7 - 2y)$	**2.**	$2x^2 - 5x - 3 > 0$	**3.**	$x(2x + 1) \leq x^2 + 6$

E1 Triangle Geometry

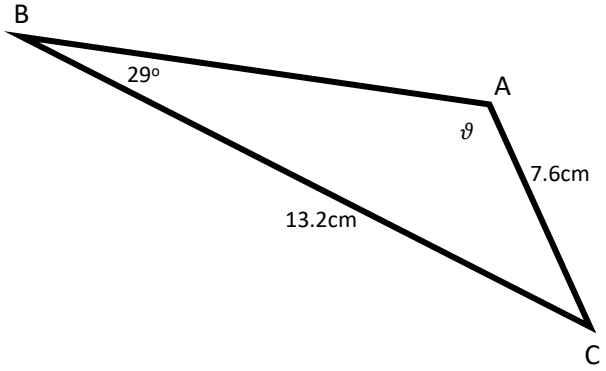
1. Calculate the length AB



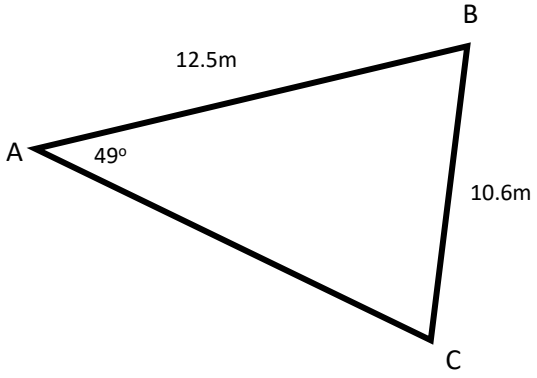
2. Calculate the angle ϑ



3. Calculate the length AB and the obtuse angle ϑ



4. Calculate the area of the triangle ABC



Practice Test 2

B1 Indices

1. $\left(3\frac{3}{8}\right)^{-1/3} = \left(\frac{27}{8}\right)^{-\frac{1}{3}}$ M1

$= \left(\frac{8}{27}\right)^{1/3}$

$= \frac{2}{3}$ A1

2. $\frac{\sqrt{x} \times \sqrt[5]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/5}}{x^2}$ M1

$= \frac{x^{7/10}}{x^2}$ A1 $= x^{-13/10}$ A1

3. $3^{3x-2} = \sqrt[3]{9}$

$3^{3x-2} = (3^2)^{1/3}$ M1

$3^{3x-2} = 3^{2/3}$

$3x - 2 = \frac{2}{3}$ M1

$3x = \frac{8}{3} \Rightarrow x = \frac{8}{9}$ A1

4. $\left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{8}\right)^{2x}$

$(2^{-1})^{1-x} = (2^{-3})^{2x}$ M1

$2^{-1+x} = 2^{-6x}$

$-1 + x = -6x$ M1

$7x = 1$

$x = 1/7$ A1

11

B2 Surds

1. $\sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$ A1

2. $(7 - 3\sqrt{5})(3\sqrt{5} - 2)$

$= 21\sqrt{5} - 14 - 45 + 6\sqrt{5}$ M1 A1

$= 27\sqrt{5} - 59$ A1

3. $\frac{7}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ M1

$= \frac{7\sqrt{3}}{15}$ A1

4. $\frac{3 + 5\sqrt{11}}{7 - \sqrt{11}} \quad \frac{(7+\sqrt{11})}{(7+\sqrt{11})}$ M1

$= \frac{21 + 3\sqrt{11} + 35\sqrt{11} + 55}{49 - 11}$ A1 A1

$= \frac{76 + 38\sqrt{11}}{38}$

$= 2 + \sqrt{11}$ A1

10

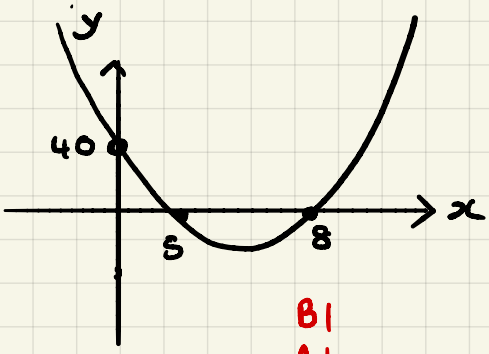
B3 Quadratics

1. (a) (i) $x^2 - 13x + 40 = 0$

$(x - 8)(x - 5) = 0$ M1

$x = 8 \quad x = 5$ A1

(ii) $y = x^2 - 13x + 40$



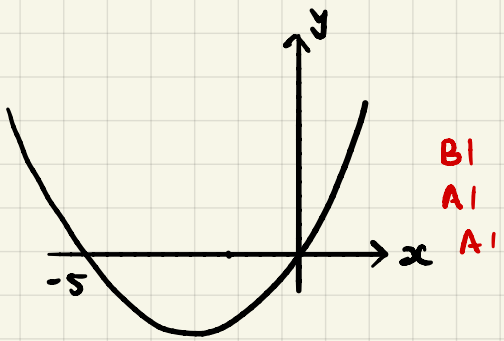
B1
A1
A1

(b) (i) $x^2 + 5x = 0$

$x(x + 5) = 0$ M1

$x = 0 \quad x = -5$ A1

(ii) $y = x^2 + 5x$



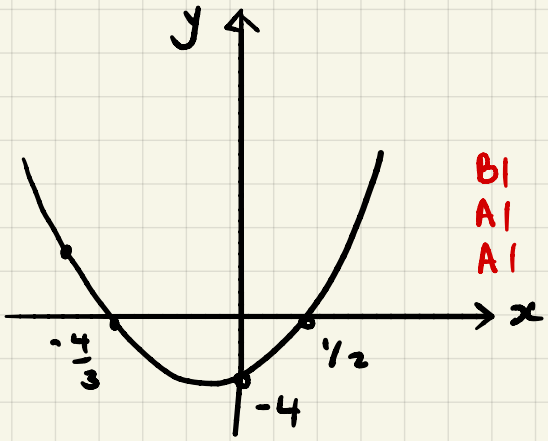
B1
A1
A1

(c) (i) $6x^2 + 5x - 4 = 0$

$(3x + 4)(2x - 1) = 0$ M1

$x = -4/3 \quad x = 1/2$ A1

(ii) $y =$



B1
A1
A1

15

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

2. (a) (i) $x^2 + 2x - 20 = 0$

$(x+1)^2 - 1 - 20 = 0$ M1

$(x+1)^2 = 21$

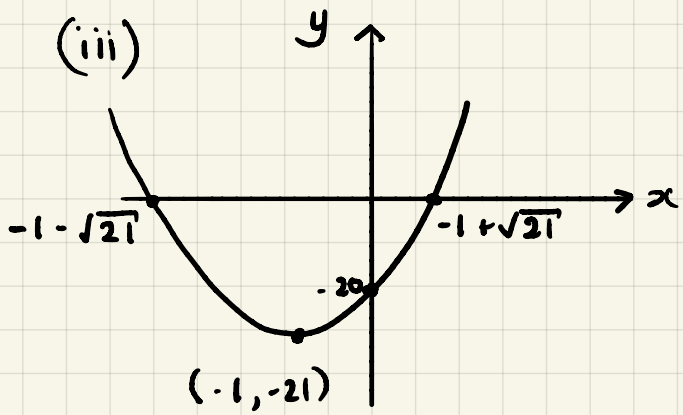
$x+1 = \pm\sqrt{21}$

$x = -1 \pm \sqrt{21}$ A1

(ii) $y = x^2 + 2x - 20$

$y = (x+1)^2 - 21$ B1

(iii)



Graphs
- B1 Shape
- A1 Vertex
- A1 intersections x-axis
- A1 intersections y-axis

(b) (i) $-11 + 8x - x^2 = 0$

$-(x^2 - 8x + 11) = 0$ M1

$-[(x-4)^2 - 16 + 11] = 0$ M1

$5 - (x-4)^2 = 0$

$(x-4)^2 = 5$

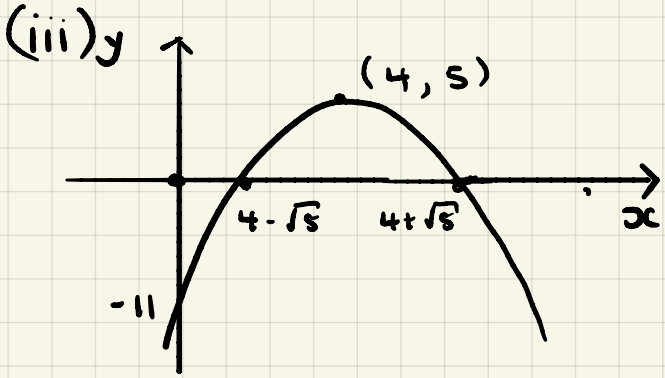
$x - 4 = \pm\sqrt{5}$

$x = 4 \pm \sqrt{5}$ A1

(ii) $y = -11 + 8x - x^2$

$y = 5 - (x-4)^2$ B1

(iii)



(c) (i) $3x^2 - 18x + 2 = 0$

$3[x^2 - 6x + \frac{2}{3}] = 0$ M1

$3[(x-3)^2 - 9 + \frac{2}{3}] = 0$ M1

$3[(x-3)^2 - \frac{25}{3}] = 0$

$3(x-3)^2 - 25 = 0$

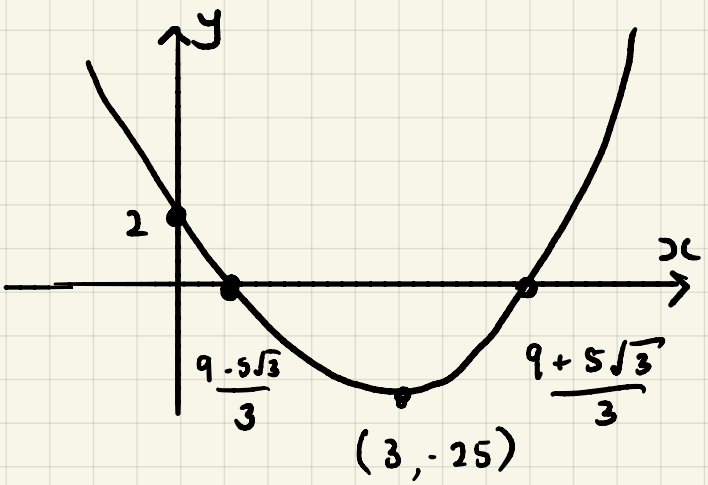
$3(x-3)^2 = 25$

$x - 3 = \pm\frac{5}{\sqrt{3}}$

$x = \frac{9 \pm 5\sqrt{3}}{3}$ A1

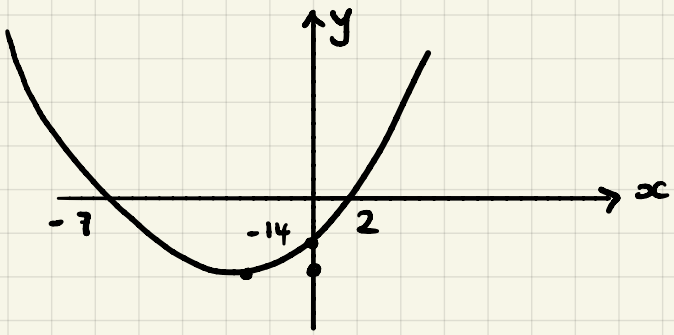
(ii) $y = 3x^2 - 18x + 2$

$y = 3(x-3)^2 - 25$ B1



23

3. (a)



$y = k(x+7)(x-2)$ M1

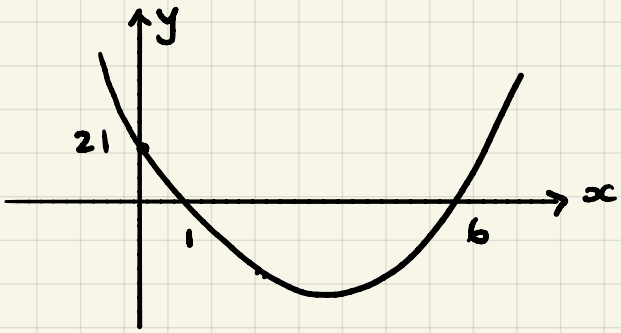
$-14 = k(7)(-2)$

$k = 1$ A1

$y = (x+7)(x-2)$

$y = x^2 + 5x - 14$ A1

(b)



$y = k(x-1)(x-6)$ M1

$21 = k(-1)(-6)$

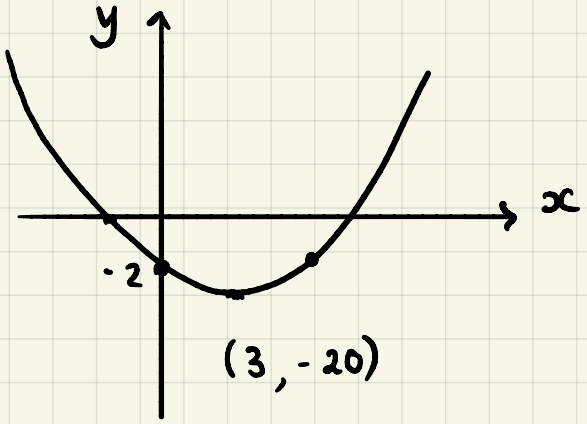
$\Rightarrow k = \frac{21}{6} = \frac{7}{2}$ A1

$y = \frac{7}{2}(x-1)(x-6)$

$y = \frac{7}{2}(x^2 - 7x + 6)$

$y = \frac{7x^2}{2} - \frac{49x}{2} + 21$ A1

(c)



$y = k(x-3)^2 - 20$ M1

$-2 = k(-3)^2 - 20$

$18 = k(9)$

$k = 2$ A1

$y = 2(x-3)^2 - 20$

$y = 2(x^2 - 6x + 9) - 20$

$y = 2x^2 - 12x - 2$ A1

9

B4. Simultaneous equations

1. $3x - 4y = 16$ $\quad 9x - 12y = 48$

$2x + 12y = 7$ $\quad 2x + 12y = 7$ M1

$11x = 55$

$x = 5$ A1

$3x - 4y = 16$

$15 - 4y = 16$

$-1 = 4y$

$y = -1/4$

$x = 5, y = -1/4$ A1

2. $3y = 2x - 8 \Rightarrow 2x = 3y + 8$

$4x + y = -5$ $\quad 4x = 6y + 16$ M1

$6y + 16 + y = -5$

$7y = -21$

$y = -3$

$2x = 3y + 8$

$2x = 3(-3) + 8$

$x = -1/2$ A1 $\quad x = -1/2$ $y = -3$ A1

3. $3x^2 - xy + y^2 = 36$

$x - 2y = 10 \Rightarrow x = 2y + 10$

$3(2y+10)^2 - (2y+10)y + y^2 = 36$ M1

$3(4y^2 + 40y + 100) - y(2y+10) + y^2 = 36$

$12y^2 + 120y + 300 - 2y^2 - 10y + y^2 = 36$

$11y^2 + 110y + 264 = 0$

$y^2 + 10y + 24 = 0$ A1

$(y+6)(y+4) = 0$

M1

$y = -6$ $\quad y = -4$

$x = 2(-6) + 10$ $\quad x = 2(-4) + 10$

$x = -2$ $\quad x = 2$

$x = -2, y = -6$ A1 $\quad x = 2, y = -4$ A1

11

B5 Inequalities

1. $4(5-2y) \geqslant 3(7-2y)$

$20 - 8y \geqslant 21 - 6y$ M1

$-1 \geqslant 2y$

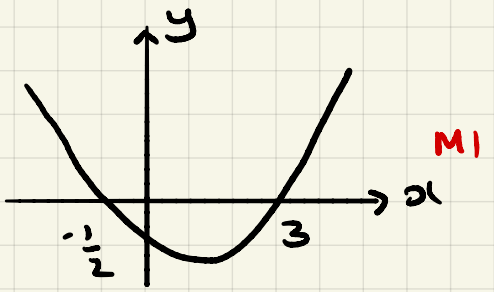
$-\frac{1}{2} \geqslant y$

$y \leqslant -\frac{1}{2}$ A1

2. $2x^2 - 5x - 3 > 0$

$(2x+1)(x-3) > 0$ M1

CVs $x = -\frac{1}{2}$ $x = 3$ A1



M1

$x < -\frac{1}{2}$ or $x > 3$ A1

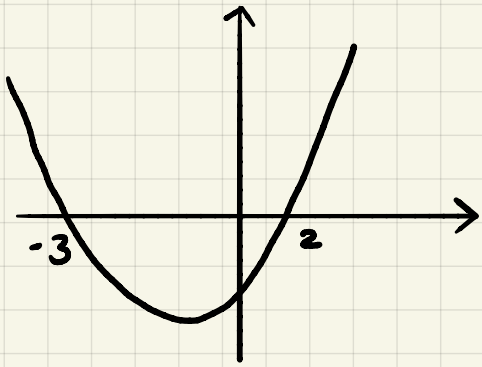
3. $x(2x+1) \leqslant x^2 + 6$

$2x^2 + x \leqslant x^2 + 6$ M1

$x^2 + x - 6 \leqslant 0$

$(x+3)(x-2) \leqslant 0$ M1

CVs $x = -3$ $x = 2$ A1



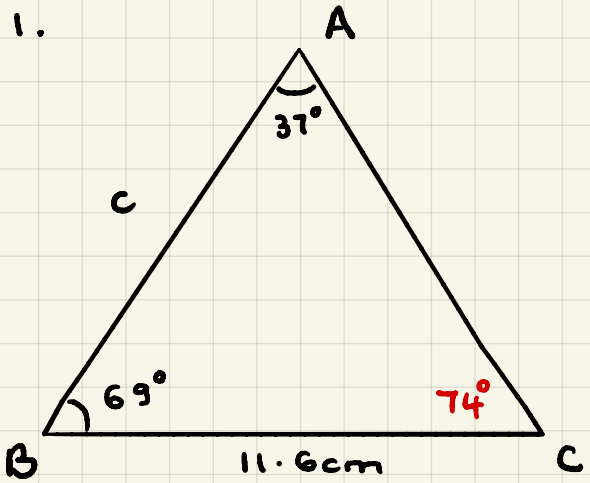
M1

$-3 \leqslant x \leqslant 2$ A1

III

E1 Triangle Geometry

1.

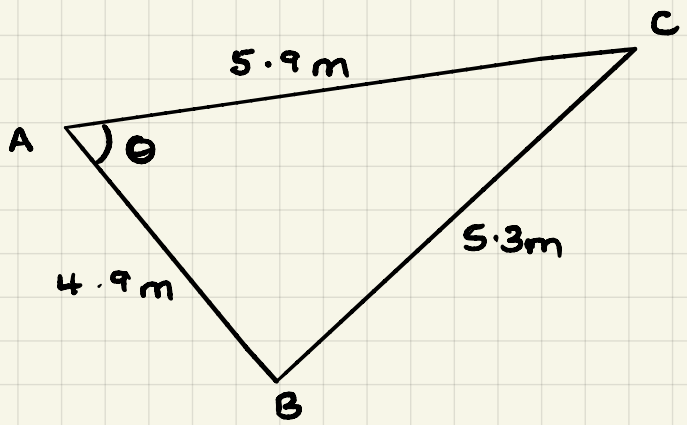


$$\frac{c}{\sin 74^\circ} = \frac{11.6}{\sin 37^\circ}$$

$$c = \frac{11.6 \sin 74^\circ}{\sin 37^\circ} \quad \text{M1}$$

$$c = 18.5\text{cm} \quad \text{A1}$$

2.



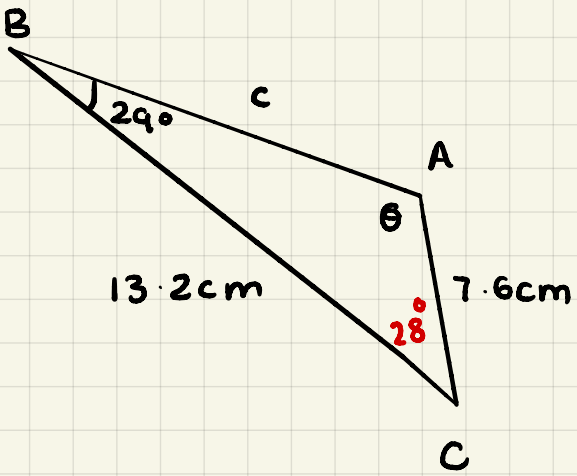
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{5.9^2 + 4.9^2 - 5.3^2}{2(5.9)(4.0)} \quad \text{M1}$$

$$\cos\theta = 0.53148$$

$$\theta = 57.9^\circ \quad \text{A1}$$

3.



θ

$$\frac{\sin\theta}{13.2} = \frac{\sin 29}{7.6}$$

$$\sin\theta = \frac{13.2 \sin 29^\circ}{7.6} \quad \text{M1}$$

$$\sin\theta = 0.8420$$

$$\theta = 57.4^\circ$$

obtuse $\Rightarrow \theta = 123^\circ$ A1

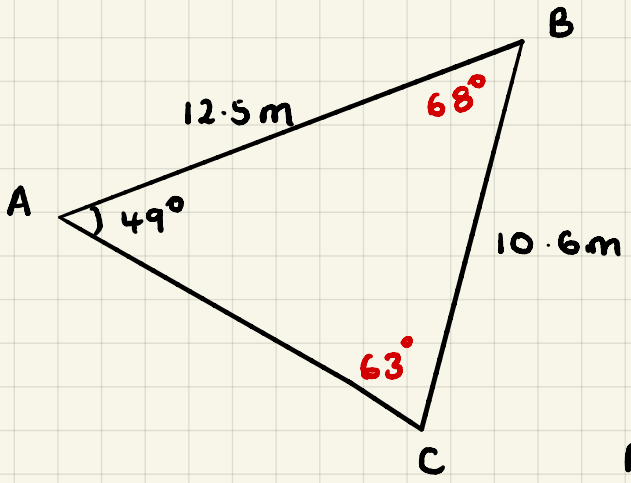
AB

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 13.2^2 + 7.6^2 - 2(13.2)(7.6)\cos 28 \quad \text{M1}$$

$$c^2 = 54.8 \Rightarrow c = 7.4\text{cm} \quad \text{A1}$$

4.



$$\frac{\sin C}{12.5} = \frac{\sin 49^\circ}{10.6}$$

$$\sin C = \frac{12.5 \sin 49}{10.6} \quad \text{M1}$$

$$C = 63^\circ \quad \text{A1}$$

$$\text{Area} = \tfrac{1}{2}\, ab \sin C$$

$$\text{Area} = \tfrac{1}{2}\,(12.5)(10.6)\sin 68^\circ \quad \text{M1}$$

$$\text{Area} = 61.5\ m^2 \quad \text{A1}$$

12